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OF THE

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VOL. 69 JANUARY, 1943 NO. 1

TECHNICAL PAPERS

AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

RIVETED AND PIN-CONNECTED JOINTS OF STEEL AND ALUMINUM ALLOYS

BY LEON S. MOISSEIFF,[1] M. AM. SOC. C. E., AND E. C. HARTMANN,[2] AND R. L. MOORE,[3] ASSOC. MEMBERS, AM. SOC. C. E.

SYNOPSIS

Development of low alloy steels and high-strength aluminum alloys, and their utilization in structures, have called the attention of engineers to the somewhat lower ductilities of these materials, as compared to carbon steel, and also to their generally higher ratios of yield strength to ultimate strength. Apprehension has been felt that these differences in physical properties would affect, unfavorably, the distribution of stress in members built of these materials, especially at connections. This belief is reflected by the fact that, in some instances, organizations responsible for design specifications, both in the United States and abroad, have set an upper limit for the ratio of yield strength to ultimate strength.

In recent years substantial advances have been made in the metallurgy of metals and in the production of numerous metallic alloys of various physical qualities. The utilizations of the metals of higher qualities have led engineers to scrutinize, more critically, the meaning and value of the information obtained from generally accepted standard tests. Along with this advance came a more thorough inquiry into the significance of the terms commonly used in characterizing and classifying metals. The old established values came to be questioned: Is there a true elastic limit? What is the true meaning of yield strength? What is ductility, and does elongation determine its value? How much ductility is needed for a given purpose, and what is its value?

In view of the practical importance of the problem and the need for experimental data concerning the behavior of these newer structural materials, it

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 1, 1943.**

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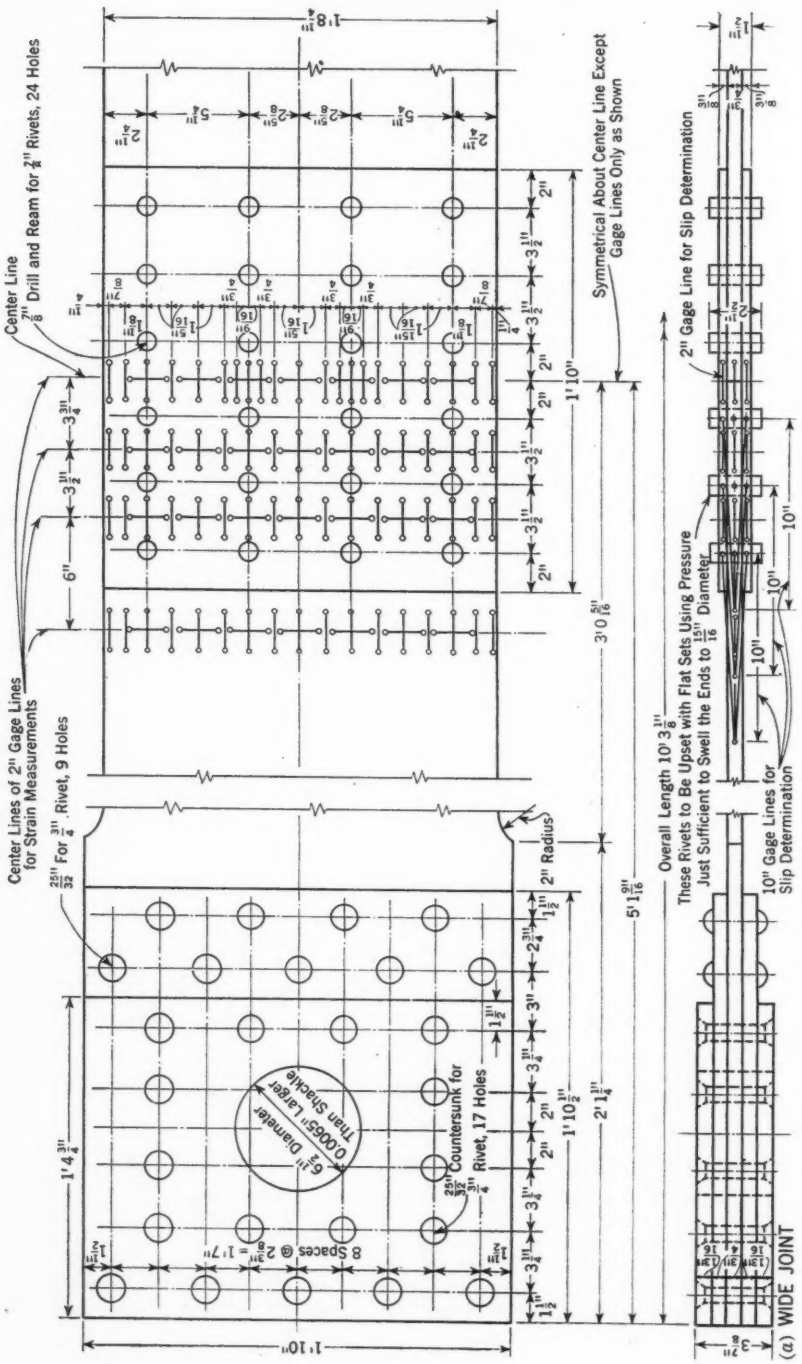

Center Line
⅞" Drill and Ream for ¾" Rivets, 24 Holes
Center Lines of 2" Gage Lines for Strain Measurements
25/32" For ¾" Rivet, 9 Holes
Symmetrical About Center Line Except Gage Lines Only as Shown
2" Gage Line for Slip Determination
Overall Length 10' 3⅛"
These Rivets to Be Upset with Flat Sets Using Pressure Just Sufficient to Swell the Ends to 15/16" Diameter
10" Gage Lines for Slip Determination
2" Radius
8 Spaces @ 2⅛" = 1'7"
0.0625" Larger Than Shackle
Countersunk for ¾" Rivet, 17 Holes
(a) WIDE JOINT

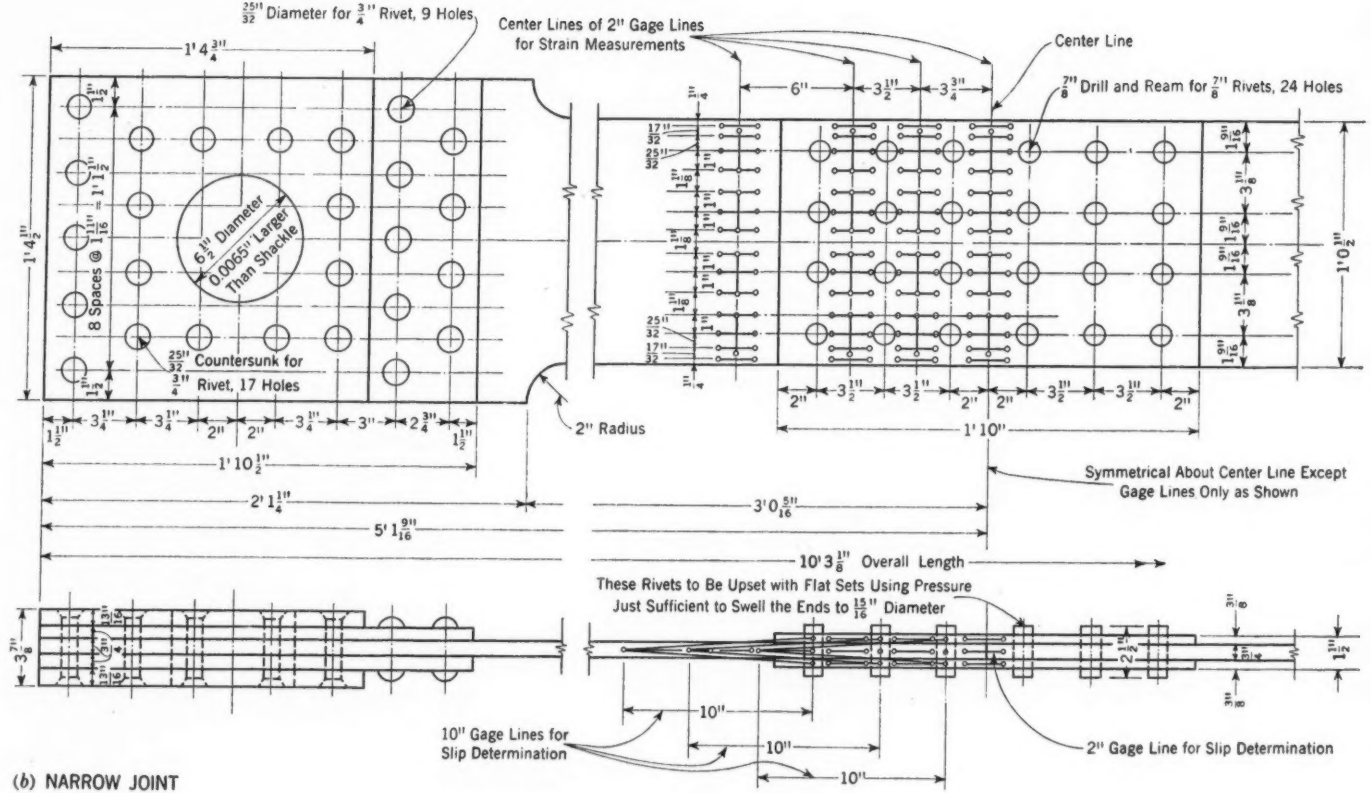
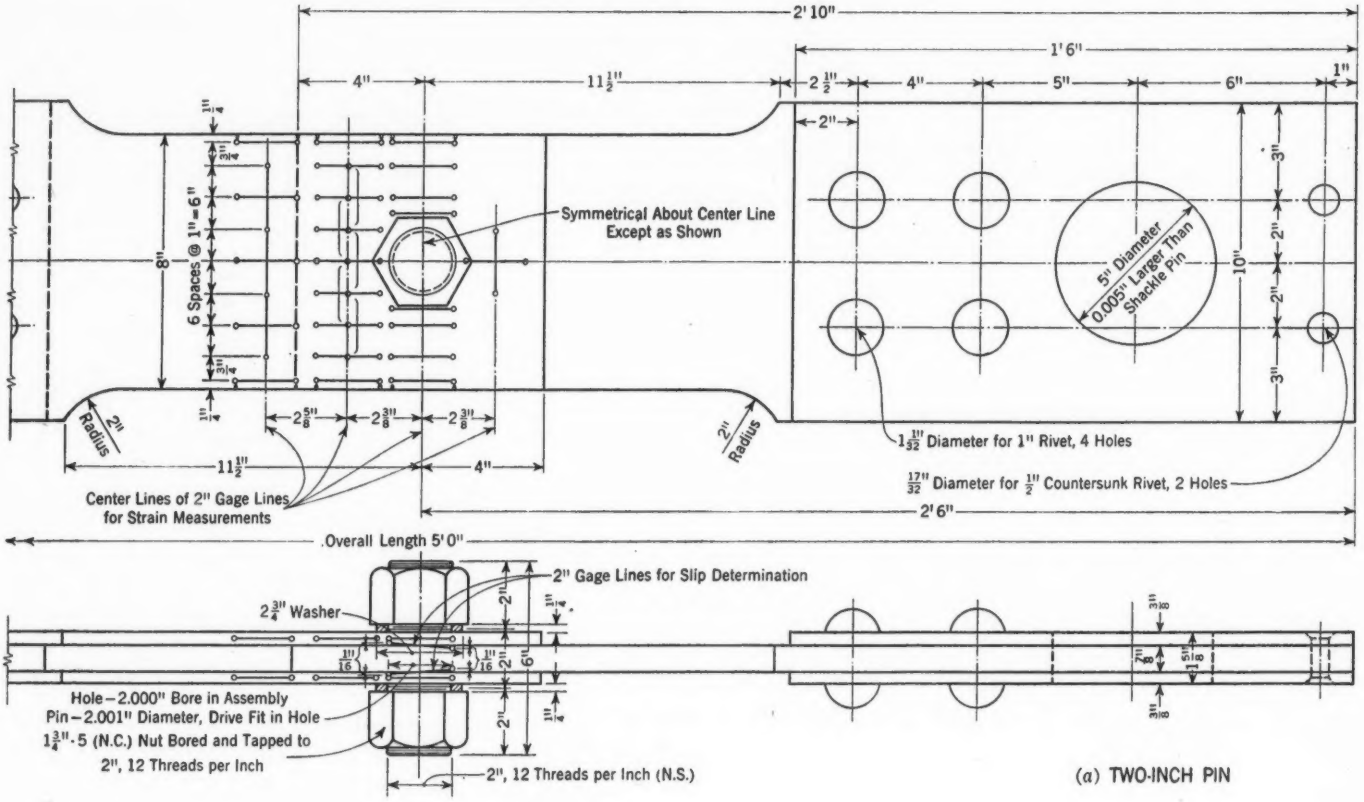
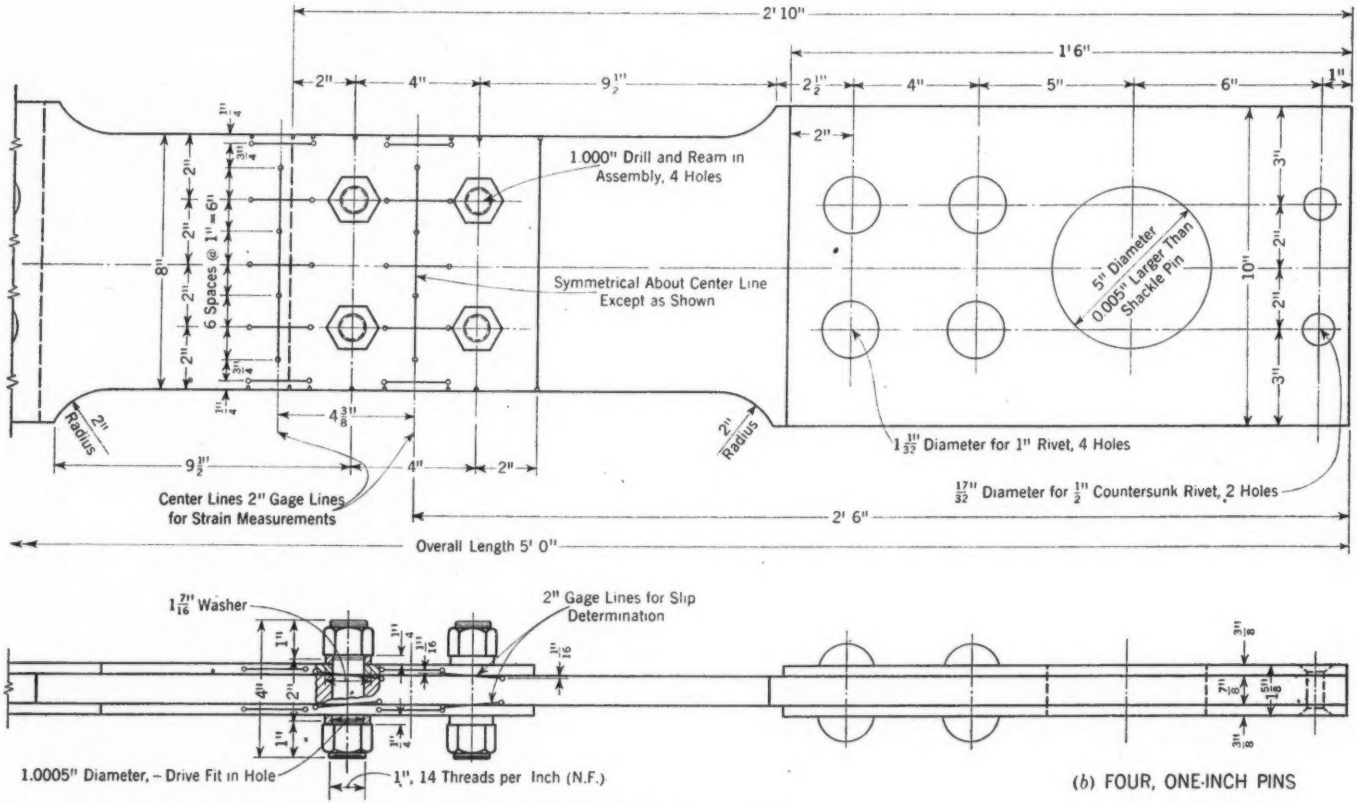

25/32" Diameter for 3/4" Rivet, 9 Holes
Center Lines of 2" Gage Lines for Strain Measurements
Center Line
7/8" Drill and Ream for 7/8" Rivets, 24 Holes
1'4 3/4"
1'4 1/2"
8 Spaces @ 2 1/16" = 1'1 1/2"
6 1/2" Diameter 0.0065" Larger Than Shackle
25/32" Countersunk for 3/4" Rivet, 17 Holes
2" Radius
1'10"
1'10 1/2"
Symmetrical About Center Line Except Gage Lines Only as Shown
2'1 1/4"
3'0 5/16"
5'1 9/16"
10'3 1/8" Overall Length
These Rivets to Be Upset with Flat Sets Using Pressure Just Sufficient to Swell the Ends to 15/16" Diameter
10" Gage Lines for Slip Determination
2" Gage Line for Slip Determination
10"
(b) NARROW JOINT

FIG. 1.—TEST JOINTS WITH TWENTY-FOUR, ¾-IN. RIVETS



(a) TWO-INCH PIN



(*b*) FOUR, ONE-INCH PINS

FIG. 2.—PIN-CONNECTED JOINTS

was thought best to undertake a program of tests of riveted and pin-connected joints. Five metals—two steels and three aluminum alloys—having a fairly wide range of ductilities and ratios of yield strength to ultimate strength, were included. All the joints tested behaved in a manner consistent with the basic assumptions of design and with the properties of the materials. Furthermore, the tests showed that the plastic yielding necessary to effect a uniform distribution of load among the rivets must be relatively small, and that differences in ratios of yield strength to ultimate strength, elongation values, and moduli of elasticity were not significant factors as far as static ultimate strengths were concerned.

Introduction

It is a fact well known to engineers and demonstrated by numerous investigators[4] that within the elastic range the load on a multiple-riveted joint is not uniformly distributed between the rivets. The basis for the usual assumption of equal shear per rivet in design is that, in ductile materials, the yielding which occurs at the points of highest stress effects a redistribution of load which becomes essentially uniform. Numerous tests on riveted joints have demonstrated the validity of this assumption as far as ultimate static strengths are concerned.[5,6]

The increasing use of alloy steels and high-strength aluminum alloys in the structural field has raised a question regarding the possible effect of the reduced ductility of these materials upon the load distribution and the strength of multiple-riveted joints. Ductility is an essential requirement of a structural material, but the limits within which it may be varied without appreciably influencing the behavior of large riveted joints are not known. In the case of the aluminum alloys, the belief has been expressed by some that the effect of reduced ductility may be offset somewhat by the low modulus of elasticity.

The tests described in this paper were undertaken in 1939, in an effort to obtain information regarding load distribution and the strength and general behavior of multiple-riveted joints involving various combinations of steel and aluminum alloys. An attempt was made to keep the specimen proportions within the limits of good design practice, and at the same time to provide a fairly wide range of strength, stiffness, and ductility. In addition to the multiple-riveted joints, a series of pin-connected joints of the same materials was also included. The scope of the program limited the tests to three rows of rivets on each side of the joint.

Description of Joints

Five different types of joints were fabricated from each of the combinations of material investigated. Fig. 1(*a*) shows the dimensions of the wide double-

[4] "Work of Rivets in Riveted Joints," by A. Hrennikoff, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 437–489, especially pp. 464–469.

[5] "Tests of Joints in Wide Plates," by W. M. Wilson, James Mather, and C. O. Harris, *Bulletin No. 239*, Univ. of Illinois Eng. Experiment Station, 1931.

[6] "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 1193–1299.

TABLE 1.—Elements of Test Joints

(Dimensions Are Inches and Square Inches)

No.	Description	Plate Material[a]				
		Silicon	Carbon	27S-T	17S-T	53S-T
1	Rivet Material	Carbon	Carbon	Carbon	17S-T	53S-W
	(*a*) Joint Designed to Fail in Shear (See Fig. 1(*a*)); Specimen 20¼ In. Wide, with 24, ¾-In. Rivets; Shear Area, 14.43 Sq In.[b]					
	Thickness, in Inches:					
2	Main plates	0.750	0.773	0.740	0.780	0.760
3	Cover plates	0.380	0.380	0.389	0.373	0.368
	Mimimum Areas, in Square Inches:					
4	Gross tension	15.20	15.38	14.98	15.10	14.90
5	Net tension[b]	12.55	12.72	12.40	12.50	12.32
6	Bearing[b]	7.88	7.98	7.77	7.83	7.72
	(*b*) Joint Designed to Fail in Tension (See Fig. 1(*b*)); Specimen 12½ In. Wide, with 24, ¾-In. Rivets; Shear Area, 14.43 Sq In.[b]					
	Thickness, in Inches:					
7	Main plates	0.750	0.755	0.740	0.780	0.760
8	Cover plates	0.380	0.380	0.389	0.373	0.368
	Minimum Areas, in Square Inches:					
9	Gross tension	9.37	9.44	9.25	9.32	9.20
10	Net tension[b]	6.75	6.80	6.66	6.71	6.62
11	Bearing[b]	7.88	7.93	7.77	7.84	7.73
	(*c*) Auxiliary Control Specimen Designed to Fail in Shear; Specimen 5 In. Wide, with 4, ¾-In. Rivets[c]; Shear Area, 2.41 Sq In.[b]					
	Thickness, in Inches:					
12	Main plates	0.750	0.750	0.750	0.750	0.750
13	Cover plates	0.375	0.375	0.375	0.375	0.375
	Minimum Areas, in Square Inches:					
14	Gross tension	3.75	3.75	3.75	3.75	3.75
15	Net tension[b]	3.09	3.09	3.09	3.09	3.09
16	Bearing[b]	1.31	1.31	1.31	1.31	1.31
	(*d*) Joint Designed to Fail in Bearing (See Fig. 2(*a*)); Specimen 8 In. Wide, with One, 2-In. Pin; Shear Area, 6.28 Sq In.[b]					
	Thickness, in Inches:					
17	Main plates	0.875	0.875	0.875	0.875	0.875
18	Cover plates	0.379	0.382	0.382	0.369	0.368
	Minimum Areas, in Square Inches:					
19	Gross tension	6.06	6.11	6.11	5.90	5.89
20	Net tension[b]	4.54	4.58	4.58	4.42	4.42
21	Bearing[b]	1.52	1.53	1.53	1.48	1.47
	(*e*) Joint Designed to Fail in Shear (See Fig. 2(*b*)); Specimen 8 In. Wide, with Four, 1-In. Pins[d]; Shear Area, 6.28 Sq In. (Except 53S-T, which Was 7.09 Sq In.)[b]					
	Thickness, in Inches:					
22	Main plates	0.875	0.875	0.875	0.875	0.875
23	Cover plates	0.379	0.382	0.382	0.369	0.368
	Minimum Areas, in Square Inches:					
24	Gross tension	6.06	6.11	6.11	5.90	5.89
25	Net tension[b]	4.54	4.58	4.58	4.42	4.32
26	Bearing[b]	3.03	3.06	3.06	2.95	3.13

[a] 27S-T, 17S-T, and 53S-T are symbols of American Society for Testing Materials (ASTM-E8-33).
[b] Areas indicated are based on the nominal hole diameters shown in Figs. 1 and 2.
[c] Specimens of each alloy in this group were also fabricated with two, ¾-in. rivets.
[d] The diameter of the 53S-T pins in Table 1(*e*) is 1 1/16 in.

strap butt joints having twenty-four $\frac{7}{8}$-in. rivets designed to fail in shear. The longitudinal spacing of the rivets was governed largely by the space required for the operation of a 2-in. Berry strain gage between rivet rows, although it is also consistent with design practice for the size of rivets and thicknesses of plate involved. Fig. 1(*b*) shows the dimensions of the narrow double-strap butt joints, also having twenty-four $\frac{7}{8}$-in. rivets, but designed to fail by tension in the plates. These joints were similar to the wide joints shown in Fig. 1(*a*) except for the over-all width and transverse spacing of rivets.

Small auxiliary double-strap butt joints, 5 in. wide and having two or four $\frac{7}{8}$-in. rivets, were designed to serve as control specimens for determining the shear strength of the driven rivets. The plate thicknesses were the same as shown in Fig. 1(*a*). Three of these specimens were provided for each combination of material, two being fabricated with four rivets (two in each half), whereas the third contained only two rivets.

Fig. 2(*a*) shows the dimensions of the joints having a single 2-in. pin, designed to fail by bearing in the outside cover plates. Fig. 2(*b*) shows the dimensions of the joints having four 1-in. pins designed to fail in shear. In one instance, it was necessary to make two sets of pins in order to obtain the close fit desired, and pins of $1\frac{1}{16}$-in. diameter were finally used.

The rivets used in the joints shown in Fig. 1 were not conventional ones, but consisted of $2\frac{1}{2}$-in. lengths of $\frac{7}{8}$-in. rod which were pressed into $\frac{7}{8}$-in. drilled and reamed holes and then upset cold in an 80-ton yoke-type riveter to approximately $\frac{1}{16}$ in. at the ends. This type of rivet was used to permit an inspection of the plate material adjacent to the rivets during the test, to provide additional clearance for taking strain readings between rivets, and to minimize the uncertain effect of friction.

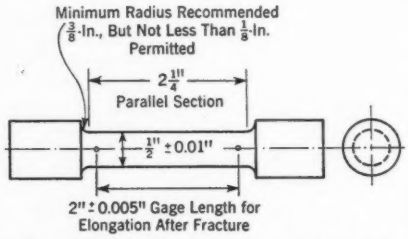


FIG. 3.—STANDARD ROUND TENSION TEST SPECIMEN WITH 2-IN. GAGE LENGTH

The pins used in the joints shown in Fig. 2 were turned oversize by approximately 0.0005 in. per in. diameter to provide a press fit in the joints. All holes in both riveted and pin-connected joints were drilled and reamed in assembly. Table 1 gives a summary of the dimensions of all the joints as well as the areas used in computing shear, bearing, and tensile stresses.

Materials.—The following combinations of material, providing a fairly wide range of ductilities and ratios of yield strength to ultimate strength, were investigated in each of the foregoing types of joints:

1. Silicon steel plates with carbon steel rivets or pins;
2. Carbon steel plates with carbon steel rivets or pins;
3. Aluminum alloy 27S-T plates with carbon steel rivets or pins;
4. Aluminum alloy 17S-T plates with 17S-T rivets or pins; and
5. Aluminum alloy 53S-T plates with 53S-W rivets or pins.

In the foregoing list, 27S-T, 17S-T, and 53S-T are specification symbols of the American Society for Testing Materials.[7] All specimens were proportional to that shown in Fig. 3.[8]

Table 2 gives a summary of the tensile properties of the plate material. As will be noted, there is some variation between the properties of different

TABLE 2.—TENSILE PROPERTIES OF PLATE MATERIAL

No.	Material	Nominal thickness (in.)	Diameter of specimen (in.)	Orientation of test specimen[a]	Pounds per Square Inch		Ratio:	Percentages	
					Yield strength[b]	Ultimate strength	Col. 5 / Col. 6	Elongation in four diameters	Reduction of area
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Silicon Steel	⅜	0.250	Normal	64,000	94,700	0.68	24.0	43
2		⅜	0.250	Parallel	64,100	95,100	0.67	29.0	57
3		¾	0.500	Normal	45,900	84,400	0.54	24.5	43
4		¾	0.500	Parallel	49,900	88,800	0.56	26.5	53
5	Carbon Steel	⅜	0.250	Normal	48,100	62,000	0.78	34.0	57
6		⅜	0.250	Parallel	43,300	62,500	0.69	39.0	64
7		¾	0.500	Normal	35,700	62,000	0.58	33.0	56
8		¾	0.500	Parallel	33,500	60,200	0.56	37.0	64
9	ASTM-27S-T	⅜	0.188	Normal	46,300	62,600	0.74	14.7	24
10		⅜	0.188	Parallel	47,800	61,900	0.77	17.3	32
11		¾	0.500	Normal	45,700	59,300	0.77	11.0	20
12		¾	0.500	Parallel	46,000	58,400	0.79	12.5	30
13	ASTM-17S-T	⅜	0.188	Normal	36,800	63,000	0.58	25.3	32
14		⅜	0.188	Parallel	41,600	63,300	0.65	24.0	38
15		¾	0.500	Normal	42,600	66,400	0.64	18.0	22
16		¾	0.500	Parallel	46,800	65,800	0.71	19.0	27
17	ASTM-53S-T	⅜	0.188	Normal	30,500	39,100	0.78	24.0	48
18		⅜	0.188	Parallel	31,500	38,600	0.82	24.0	53
19		¾	0.500	Normal	36,100	40,700	0.89	14.0	35
20		¾	0.500	Parallel	35,700	39,000	0.92	17.5	50

[a] "Normal" denotes that the specimen was normal to the direction of rolling; and "parallel" denotes "parallel to the direction of rolling."
[b] For aluminum steel, the yield strength is measured by the stress corresponding to an offset of 0.2% from the initial modulus line; and, for steel, it is the stress corresponding to the drop-of-the-beam yield point.

thicknesses of the same materials, particularly in the case of yield strengths, and there is also some evidence of directional properties. Ratios of tensile yield strength to ultimate strength ranged from a minimum of 0.54 for the silicon steel to a maximum of 0.92 for the aluminum alloy 53S-T. Corresponding ratios for the carbon steel plates exceeded those for the silicon steel, the highest value in any case being 0.78, which is considerably above that generally assumed for such material.

The elongation values given in Table 2 range from a minimum of 11% for the aluminum alloy 27S-T to a maximum of 39% for the carbon steel. It should be emphasized that elongation values do not necessarily bear a definite relationship to ratios of tensile yield strength to ultimate strength; nor do they always give a complete picture of the ductility of a metal. For example, the

[7] ASTM-E8-33.

[8] "Tentative Methods of Tension Testing of Metallic Materials," Supplement to ASTM Standards, Pt. 1, 1940, E8-40T, Fig. 3, p. 454.

elongations found for the aluminum alloy 53S-T plates exceeded those for the aluminum alloy 27S-T; yet the ratios of yield strength to ultimate strength for the 53S-T alloy were considerably higher. The elongations for the aluminum alloy 17S-T plates were slightly higher than for the aluminum alloy 53S-T plates and the yield-ultimate strength ratios were lower; yet in general 17S-T alloy is not as satisfactory for forming operations as 53S-T alloy. The explanation for this behavior is indicated, at least qualitatively, by the reduction in area values included in Table 2. The values for the 53S-T alloy plates were higher than those for either of the other aluminum alloys, being more nearly comparable with those for the silicon steel. The reductions in area for the carbon steel plates were the highest observed.

No attempt was made to determine moduli of elasticity, the nominal values of 29,500,000 and 10,300,000 lb per sq in. being assumed satisfactory for interpreting the measured strains in the steel and aluminum alloys, respectively. The corresponding values of Poisson's ratio were assumed to be $\frac{1}{4}$ and $\frac{1}{3}$.

Testing Procedure

All tests, except those on the small auxiliary shear specimens, were made in the Templin precision metal-working machine having a 1,000,000-lb maximum capacity tensile range (named for R. L. Templin, M. Am. Soc. C. E.).[9] Load was applied to the large joints of the type shown in Fig. 1 through six $\frac{1}{2}$-in. pins fitted in special tension shackles. The smaller pin-connected joints shown in Fig. 2 were loaded through 5-in. pins equipped with bushings to fit in the same tension shackles. Figs. 4 and 5 are close-up views of different types of joints. The small auxiliary shear specimens were tested in a machine having a 300,000-lb maximum capacity range.

The test procedure followed on all the large joints included one or more types of slip determination, using a 10-in. Whittemore strain gage or dial indicators on 2-in. and 48-in. gage lengths; the measurement of strains by means of 2-in. Berry strain gages; and the determination of ultimate strengths. The slip readings obtained by means of the Whittemore strain gage involved establishing 10-in. gage lengths on the edges of the main plates of the joints with one gage hole at each rivet section where slip was to be determined. Single gage holes were then located on the adjacent cover plates at these same rivet sections so that readings could be taken on three lines at each section using a common gage hole at each end. The difference in readings obtained on the main and cover plates gave the slip directly, irrespective of the strain that may have occurred over the gage lengths. The angle between the gage lines was so slight that the error introduced by the difference in slopes was on the order of one part in a thousand. Check readings were held to within a tolerance of one division of the dial, which corresponded to a change in gage length of 0.0001 in. Fig. 1 shows the location of the 10-in. slip gage lines.

A dial indicator, graduated in thousandths of an inch and having a 1-in. travel, was used for the combined slip and strain determinations made on the 2-in. and 48-in. gage lengths. For the short gage length, two points were

[9] *Civil Engineering*, May, 1940, p. 323.

attached to the indicator—one a fixed point attached to the back of the dial and one a movable point attached to the stem so that it could be operated over a considerable range in the vicinity of 2 in. Readings estimated to the nearest 0.0001 in. were taken with this instrument across the gap between the main plates at the center of all the double-strap butt joints and also diagonally across the main and cover plates in the pin-connected joints at the sections through the pins. Figs. 1 and 2 show the location of these slip gage lines. For the 48-in. gage lengths (which, of course, were limited to the large joints), a dial indicator was attached to an adjustable measuring rod and readings were taken between small pins screwed into tapped holes in the edges of the main plates.

Figs. 1 and 2 show the location of the 2-in. longitudinal and transverse gage lines on which strains were investigated by Berry strain gages. It was appreciated that a complete picture of the highly localized stress distribution in the vicinity of the rivets and pins could not be obtained from the layout of the gage lines shown; yet this procedure seemed satisfactory as far as indicating general characteristics of behavior for different materials was concerned. Check strain readings were held to within a tolerance of four-tenths of one division on the dial, which corresponds to a stress of approximately 1,200 lb per sq in. in steel and 400 lb per sq in. in aluminum.

The measurement of both longitudinal and transverse strains at a number of points was necessary to determine the state of stress existing and to indicate the manner in which the strains should be interpreted. The consistent tendency for the measured transverse strains to range from $\frac{1}{4}$ to $\frac{1}{3}$ the longitudinal values, and to be of opposite sign, provided ample justification for the assumption of a unidirectional state of stress on the sections investigated. Accordingly in all tests, measured longitudinal stresses (in the direction of loading) were obtained by multiplying the unit longitudinal strains by the modulus of elasticity.

Because of the fact that the tests on a number of the joints extended over a period of several days and it was not always possible to maintain a uniform temperature, particular attention was paid to the matter of providing standard bars for all the slip and strain measuring instruments used. These reference bars, of both steel and aluminum, were kept in contact with, or near, the joints being tested at all times.

All of the joints were subjected to a number of increments of load leading to ultimate failure. The general procedure followed was to select increments of load small enough that from six to eight sets of slip determinations could be obtained below the design load. In most cases these same increments were used as long as any slip measurements were taken. Permanent sets or slips were measured for the design loads and for several increments exceeding this value.

Strain measurements were usually limited to from four to six increments of load within the elastic range of the plate material or within the limits of strain that could be covered by the gages. In most cases an effort was made to obtain a complete set of strains at loads equal to approximately 1, $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ times the design load. Intermediate as well as higher loads were used in a

FIG. 4.—CARBON STEEL JOINTS, 12½ IN. WIDE

FIG. 5.—PIN-CONNECTED JOINT, 8 IN. WIDE; 2-IN. PIN

number of cases, however, particularly on the aluminum joints. All the joints of any one type were subjected to at least one common load so that a direct comparison of strains could be made. Design stresses used as a basis for selecting increments of test load are given in Table 3.

TABLE 3.—DESIGN STRESSES (KIPS[a] PER SQUARE INCH) USED AS A BASIS FOR SELECTING TEST LOAD INCREMENTS

Type of stress	Silicon steel	Carbon steel	27S-T	17S-T	53S-T[b]
Shear	..	13.5	..	10	7[b]
Tension	24	18.0	21	16	12
Bearing	32	27.0	30	26	18

[a] 1 kip = 1 "kilo-pound" = 1,000 lb. [b] The shear specimen in this group was ASTM-53S-W.

RIVETED JOINTS DESIGNED TO FAIL IN SHEAR

Fig. 1(*a*) shows the dimensions of the large joints included in this series of tests. There are four points of interest to be considered: The distribution of load between the rivet rows; the load at which the first appreciable yielding or permanent slip between plates was observed; the distribution of measured strain in the plates; and the ultimate shear strength of the joints.

Table 4 presents data for plotting the load-slip relations for different rivet sections in the small auxiliary control specimens, having two and four rivets, and in the joints, 20¼ in. wide, having twenty-four rivets. By reducing all loads to an average value per rivet, it was possible to plot these relations for all joints of the same material in one figure, thereby facilitating direct comparisons (see Fig. 6). In the case of the auxiliary specimens having only two rivets, of course, there was no question as to the distribution of load. In the auxiliary specimens having four rivets the load should have been equally distributed between the rivets from conditions of symmetry, and the load-slip relations shown for these specimens support this contention. In the large joints having twenty-four rivets, however, there is a consistent difference between the slip obtained at sections A and C, through the end rows of rivets, and that obtained at section B through the intermediate rows. If the load on each row of rivets was proportional to the observed slip between the plates, then rows A and C were more effective than row B, which is in accordance with the theory for the behavior of such joints.[4] The fact that slip was observed in all joints beginning with the application of the first load indicates that there was no appreciable friction between the plates, which result is contrary to that reported by recent investigators using riveted joints of a more conventional type.[6,10] The relations between load and slip were not linear, even in the early stages of the tests, which is also contrary to the behavior usually assumed for riveted joints within the elastic range. There was no evidence of any slip necessary to bring the rivets into bearing, probably because the rivets in these joints were driven cold, in holes which had been drilled and reamed in assembly.

Table 5 gives a summary of the measured slips obtained on the auxiliary shear specimens with corresponding ratios of load to slip, or so-called "*K*-values." Since linear load-slip relations were not obtained for any of the joints, *K*-values have been given for the shear design loads only. Values of

[10] "Fatigue Tests of Riveted Joints," by W. M. Wilson and F. P. Thomas, *Bulletin No. 302*, Univ. of Illinois Eng. Experiment Station, 1938.

TABLE 4.—AVERAGE PLATE SLIP, IN TEN-THOUSANDTHS OF AN INCH, AS OBSERVED ON JOINTS DESIGNED TO FAIL IN SHEAR

No.	Joint[a]	AVERAGE LOAD PER RIVET, IN KIPS:											
		2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5	30.0
	(*a*) SILICON STEEL PLATES; CARBON STEEL RIVETS												
	Two-Rivet:												
1	Section A	2	2	4	6	8	10	13	18	36	90	138	195
	Four-Rivet:												
2	Section A	2	3	5	8	9	12	15	23	37	81	129	189
3	Section B	2	4	6	7	10	13	16	22	39	86	...	196
	Twenty-Four Rivet:												
4	Section A	3	4	6	9	12	16	19	26	51	95	132	195
5	Section B	1	2	2	5	7	8	11	17	42	86	...	186
6	Section C	1	3	6	9	12	14	18	25	49	93	133	195
	(*b*) CARBON STEEL PLATES; CARBON STEEL RIVETS												
	Two-Rivet:												
7	Section A	1	3	5	7	9	12	15	18	26	94	145	204
	Four-Rivet:												
8	Section A	1	3	4	6	9	11	14	18	31	91	136	195
9	Section B	1	3	5	7	10	12	16	20	33	93	141	200
	Twenty-Four Rivet:												
10	Section A	2	5	7	10	13	17	20	27	56	100	155	225
11	Section B	1	2	4	5	7	10	13	19	47	91	146	217
12	Section C	2	3	5	7	10	13	17	23	51	91	153	226
	(*c*) 27S-T PLATES; CARBON STEEL RIVETS (SEE FIG. 6)												
	Two-Rivet:												
13	Section A	3	7	11	16	21	28	35	45	69	107	158	216
	Four-Rivet:												
14	Section A	3	7	12	17	23	30	37	47	72	111	162	229
15	Section B	3	7	10	15	19	25	31	42	67	108	160	228
	Twenty-Four Rivet:												
16	Section A	4	9	15	21	27	35	45	62	92	142	207	286
17	Section B	2	4	7	11	15	20	26	37	69	120	182	260
18	Section C	4	8	14	19	26	33	43	60	91	141	210	...
	(*d*) 17S-T PLATES; 17S-T RIVETS												
	Two-Rivet:												
19	Section A	5	11	17	23	31	40	50	61	77	96	121	162
	Four-Rivet:												
20	Section A	5	11	17	23	31	40	50	62	77	96	122	164
21	Section B	5	11	17	24	32	40	50	61	76	95	121	168
	Twenty-Four Rivet:												
22	Section A	5	11	18	26	36	47	62	78	98	126	165	226
23	Section B	3	8	13	19	26	35	45	59	77	101	138	197
24	Section C	6	13	20	28	38	49	63	80	100	128	170	228
	(*e*) 53S-T PLATES; 53S-W RIVETS												
	Two-Rivet:												
25	Section A	4	10	18	28	42	64	126	...	...	...	...	...
	Four-Rivet:												
26	Section A	5	11	18	28	43	70	130	...	...	...	...	...
27	Section B	5	11	19	30	45	72	134	...	...	...	...	...
	Twenty-Four Rivet:												
28	Section A	6	13	23	35	52	85	160	...	...	...	...	...
29	Section B	3	7	13	24	40	68	145	...	...	...	...	...
30	Section C	5	11	20	31	49	82	155	...	...	...	...	...

[a] For diagrams of joints, location of sections, and typical plotting of data, see Fig. 6.

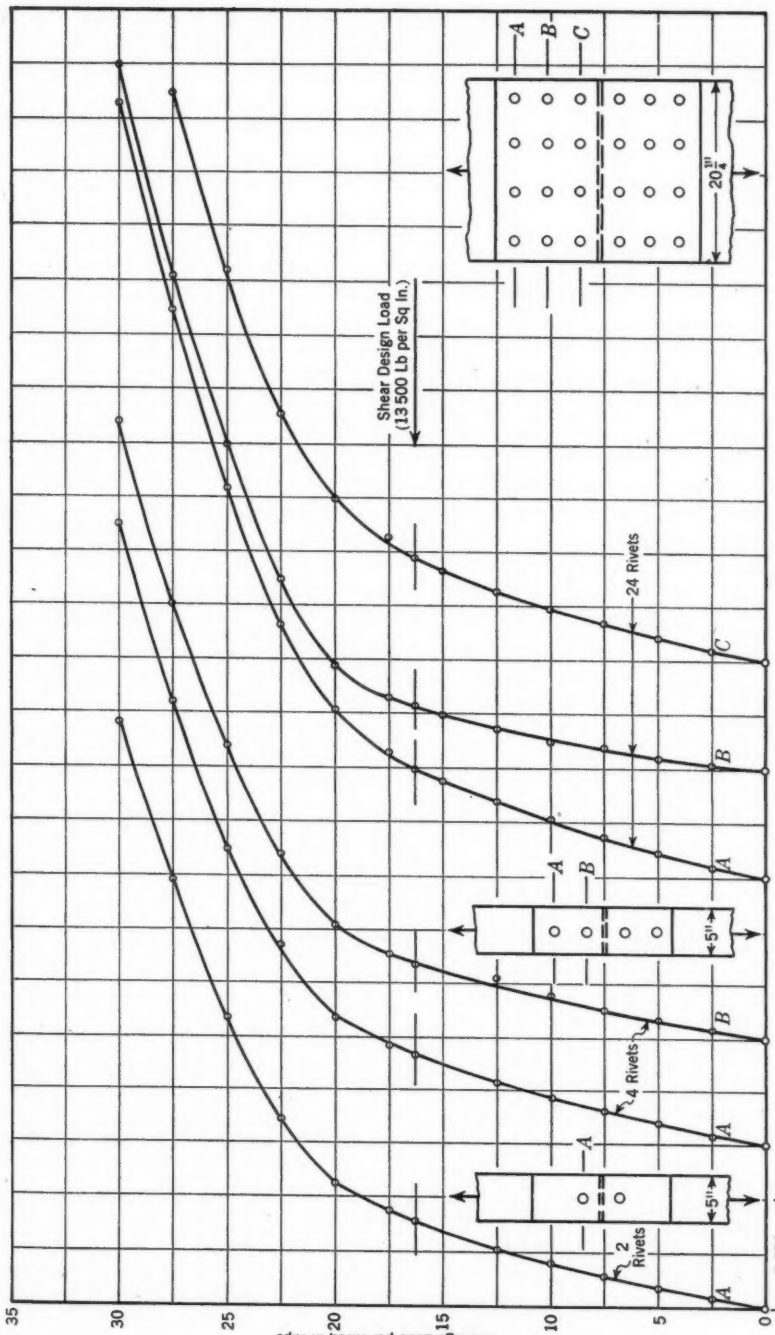


FIG. 6.—LOAD-SLIP CURVES FOR JOINTS OF 27S-T STEEL PLATES AND CARBON STEEL RIVETS

the latter are indicated on the curves in Fig. 6 and by the data in Table 4. There are several points of interest to be noted from the data given in Table 5. The measured slips for section A of the joints having only two rivets were

TABLE 5.—MEASURED PLATE SLIP IN AUXILIARY SHEAR SPECIMENS

MATERIAL		Shear design stress (lb per sq in.)	Corresponding design load per rivet (lb)	MEASURED SLIP, IN INCHES[a]				Slip modulus or K-value[b]
Plates	Rivets			Two-rivet specimen, section A	Four-rivet specimen		Average	
					Section A	Section B		
Silicon	Carbon	13,500	16,250	0.0012	0.0013	0.0014	0.0013	12,500,000
Carbon	Carbon	13,500	16,250	0.0013	0.0013	0.0014	0.0013	12,500,000
27S-T	Carbon	13,500	16,250	0.0031	0.0033	0.0027	0.0030	5,420,000
17S-T	17S-T	10,000	12,000	0.0029	0.0028	0.0030	0.0029	4,140,000
53S-T	53S-W	7,000	8,400	0.0021	0.0021	0.0022	0.0021	4,000,000

[a] Determined from load-slip curves shown in Fig. 6 and Table 4. [b] Ratio of design load per rivet to average slip.

practically the same as those measured on sections A and B of the joints having four rivets; the K-values for the joints having aluminum plates and rivets were about one third of those obtained for the all-steel specimens, which corresponds approximately to the ratio of the moduli of elasticity for these two materials; and the K-values for the joints having aluminum alloy 27S-T plates and steel rivets were about 43% of those obtained for the all-steel specimens, indicating that deformation in the aluminum plates rather than detrusion of the rivets was largely responsible for the magnitude of the slips observed.

The distribution of load between sections A, B, and C of the large joints may be determined from the ratio of the measured slips on these sections without regard to the results obtained on the auxiliary specimens where the load

TABLE 6.—MEASURED PLATE SLIP AND CORRESPONDING LOAD DISTRIBUTION IN RIVETED JOINTS 20¼ IN. WIDE DESIGNED TO FAIL IN SHEAR

MATERIAL		Design load per rivet (lb)	MEASURED SLIP IN INCHES[a]; SECTIONS:			PERCENTAGE OF TOTAL LOAD CARRIED BY SECTIONS:[b]			THEORETICAL LOAD DISTRIBUTION (PERCENTAGES)[c] AT SECTIONS:		
Plates	Rivets		A	B	C	A	B	C	A	B	C
Silicon	Carbon	16,250	0.0018	0.0009	0.0015	43	21	36	36.8	26.4	36.8
Carbon	Carbon	16,250	0.0018	0.0010	0.0014	43	24	33	36.8	26.4	36.8
27S-T	Carbon	16,250	0.0040	0.0023	0.0037	40	23	37	37.4	25.2	37.4
17S-T	17S-T	12,000	0.0034	0.0024	0.0036	36	26	38	36.7	26.6	36.7
53S-T	53S-W	8,400	0.0027	0.0016	0.0024	40	24	36	36.6	26.8	36.6

[a] Determined from load-slip curves shown in Fig. 6 and Table 4. [b] Determined from relative magnitudes of measured slips on sections A, B, and C. [c] Based on K-values given in Table 5.

distribution was known. Table 6 gives a summary of the measured slips on sections A, B, and C for the shear design loads and gives the corresponding percentages of load carried by each rivet row. These results are consistent

in indicating that the two end rows in each half of the joints carry more load than the intermediate rows. There are differences between the percentage values for sections A and C, however, which cannot be explained, other than by stating that they probably resulted from minor inequalities in fabrication. In four out of five cases the rivets on section A apparently carried from 3% to 10% more of the total load than the rivets on section C which theoretically should have been equally effective. Table 6 also gives, for comparative purposes, the theoretical distribution of load for the large joints based upon the *K*-values obtained from the auxiliary shear specimens. The agreement between these theoretical values and those determined from the slip measurements is not particularly good, although in both cases the relative ineffectiveness of rivet row B, with respect to rows A and C, is clearly indicated. On the basis of the slip measurements it appears that the differences between the ordinary assumption of uniform load distribution and that actually obtained may be greater than indicated by theoretical considerations. This observation was also made by Raymond E. Davis and Glenn B. Woodruff, Members, Am. Soc. C. E., and Harmer E. Davis, Assoc. M. Am. Soc. C. E., who reported that inequalities arising from fabrication may cause an erratic and irregular load partition in large riveted joints which is great in comparison with the differences in load partition called for by the elastic theory.[6]

Table 7 gives a summary of loads for sections A, B, and C of the wide joints computed from the measured slips for the design loads and the *K*-values ob-

TABLE 7.—COMPARISON BETWEEN DESIGN LOADS AND CORRESPONDING LOADS BASED ON MEASURED SLIPS FOR RIVETED JOINTS 20¼ IN. WIDE DESIGNED TO FAIL IN SHEAR

MATERIAL		Shear design stress (lb per sq in.)	Corresponding design load for joint (lb)	LOAD CORRESPONDING TO MEASURED SLIP[a] (LB)				Ratio: Load from slip / Design load
Plates	Rivets			Section A	Section B	Section C	Total	
Silicon	Carbon	13,500	195,000	90,000	45,000	75,000	210,000	1.08
Carbon	Carbon	13,500	195,000	90,000	50,000	70,000	210,000	1.08
27S-T	Carbon	13,500	195,000	86,700	49,800	80,200	216,700	1.11
17S-T	17S-T	10,000	144,000	56,300	39,700	59,600	155,600	1.08
53S-T	53S-W	7,000	101,000	43,200	25,600	38,400	107,200	1.06

[a] Based on *K*-values for auxiliary shear specimens given in Table 5 and measured slips for sections A, B, and C of wide joints given in Table 6.

tained from the small auxiliary specimens. The fact that the differences between the total loads obtained for the wide joints in this manner and those actually applied ranged from only 6% to 11% for all joints indicates a satisfactory correlation between the results of these related tests.

From the load-slips curves shown in Fig. 6 and from data in Table 4, it is not possible to select a well-defined point of first yielding although the "knees" in the curves for the steel joints are somewhat more pronounced than in those for the aluminum joints, as would be expected from the fundamental difference in the stress-strain characteristics of these materials. One observation which

may be made from these data, however, is that the slip showed a marked increase at loads only slightly greater than the shear design values for the rivets. Slip measurements on 2-in. gage lengths across the gap between the main plates indicated the same load-slip characteristics as far as first yielding was concerned. Fig. 7 shows a typical set of load-slip curves of this type, as well as permanent slip data for the aluminum alloy 27S-T joints with carbon steel rivets. In every case, permanent slips corresponding to the design loads were on the order of only a few ten-thousandths of an inch.

To determine the load-strain curves for the wide joints of this series, strains, combined with slip, were measured over 48-in. gage lengths. Except for the fact that these curves indicated a more definite elastic range or linear relationship between load and strain, their shape was similar to those in Fig. 7. Within the elastic range the total measured elongations varied from about 6% to 10% less than those computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500,000 and 10,300,000 lb per sq in. for steel and aluminum, respectively. This result is indicative of the tightness of the joints, where the effect of slip was more than offset by the effect of the extra plate thickness at the joint splices.

Fig. 8 shows the measured distribution of longitudinal stress in the wide joints for two increments of load, one within the range before any appreciable permanent slip had occurred and the other well above the range of first yielding. The values given are the measured unit strains multiplied by the moduli of elasticity—for steel, 29,500 kips per sq in., and for aluminum, 10,300 kips per sq in. An examination of the data indicates that essentially the same behavior was obtained for all the different combinations of materials investigated. On section A in the main plates a reasonably uniform distribution of stress was observed, as would be expected, and the averages of the measured values for all gage lines checked the average computed stresses within a few per cent. On sections B and C, between rivet rows on the cover plates, a fairly uniform distribution of stress was observed for the smaller load, but wide variations were found for the higher loads. In fact, compressive rather than tensile strains were measured in some cases on the longitudinal gage lines in line with the rivets.

Fig. 8 also shows the percentages of total applied load accounted for by the average measured cover-plate stresses on sections B and C. As will be noted from a comparison with the percentages given in Table 6, these values are not in very good agreement with the load distribution based on the slip measurements. The uncertainties apparent regarding the actual stress distribution produced in the cover plates suggest that the strain measurements as obtained were probably not very sensitive to the load distribution between rivet rows. There is no question, of course, as to the amount of load carried on section D at the middle of the joints. The distribution of stress measured at this section was quite varied, although the average values in most cases were within a few per cent of the averages computed.

It should be stated that the average measured stresses referred to herein, and throughout the remainder of this paper, are simply the arithmetic average of all the values found on the gage lines of any one section. No effort was

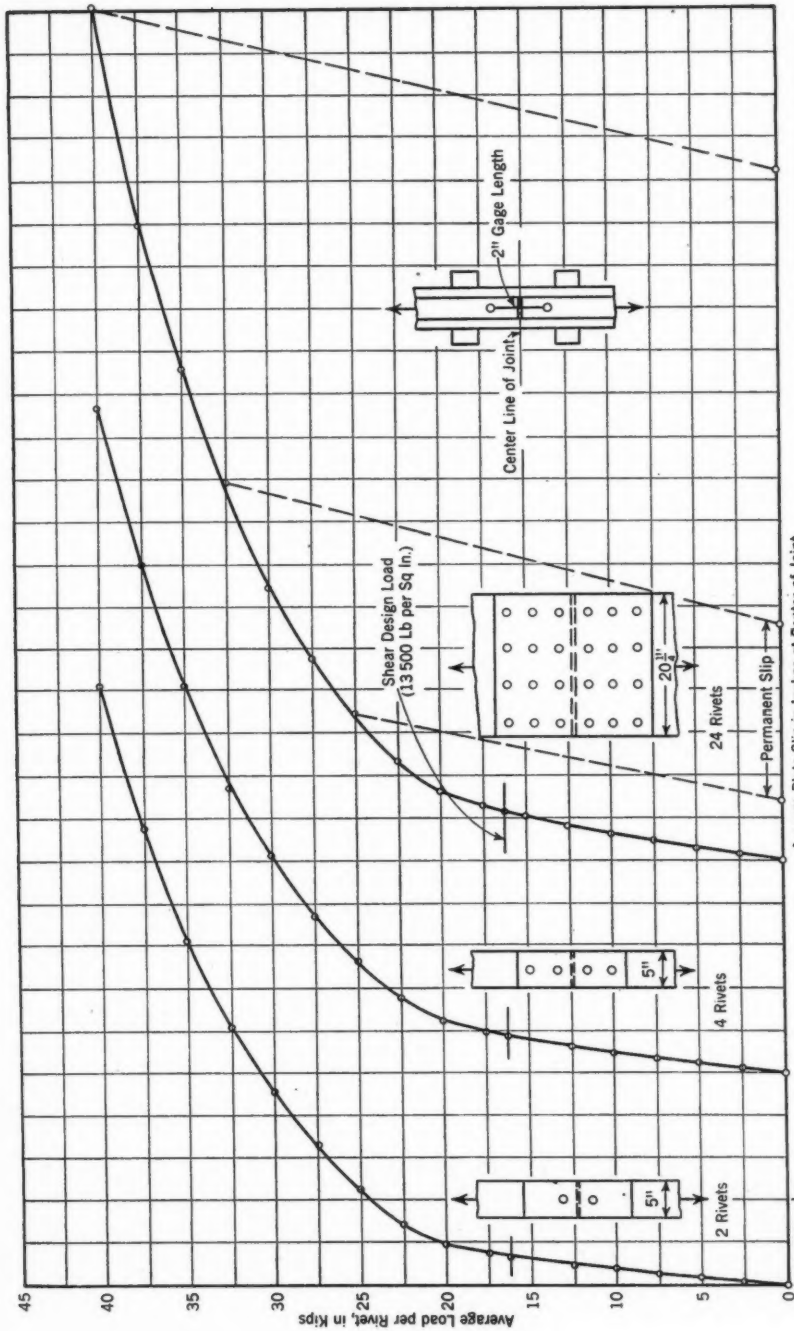


FIG. 7.—SLIP DETERMINED BY A DIAL INDICATOR ON 2-IN. GAGE LENGTHS; JOINTS OF 27S-T PLATES AND CARBON STEEL RIVETS, DESIGNED TO FAIL IN SHEAR

made to obtain "weighted" averages for the cases in which the stress distribution was quite varied and the gage lines were not equally spaced. Consequently, in a few instances a discrepancy may be noted on the distribution diagrams between the average and individual measured stresses. On sections where the stress distribution is reasonably uniform, the relative position of the gage lines is not important.

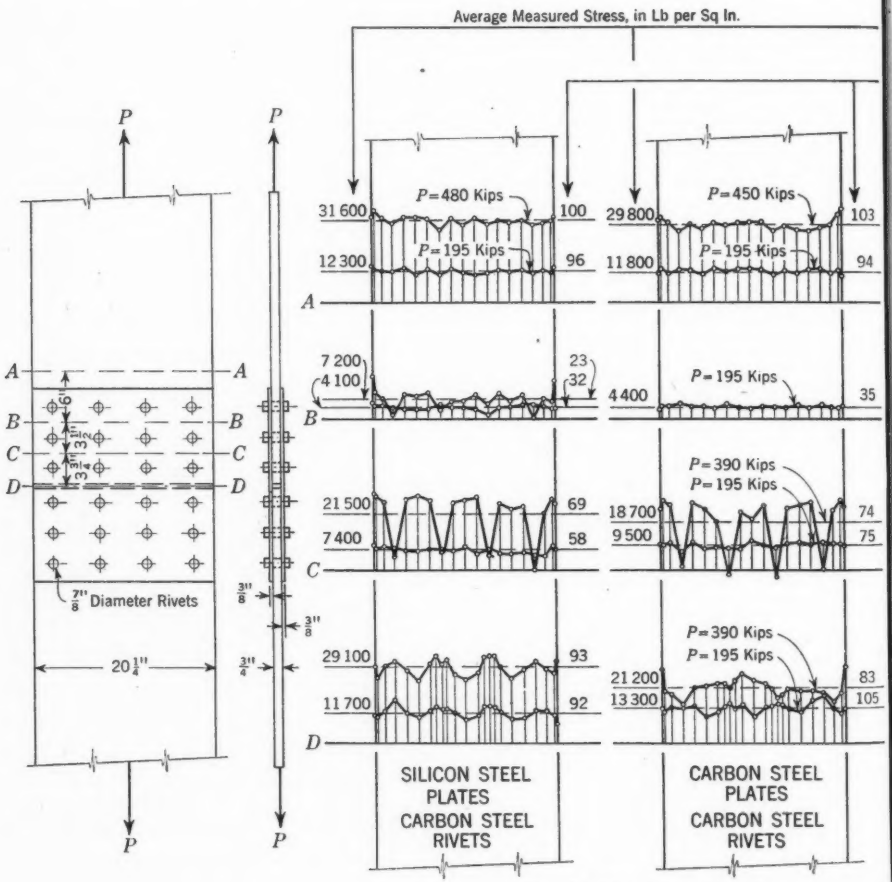
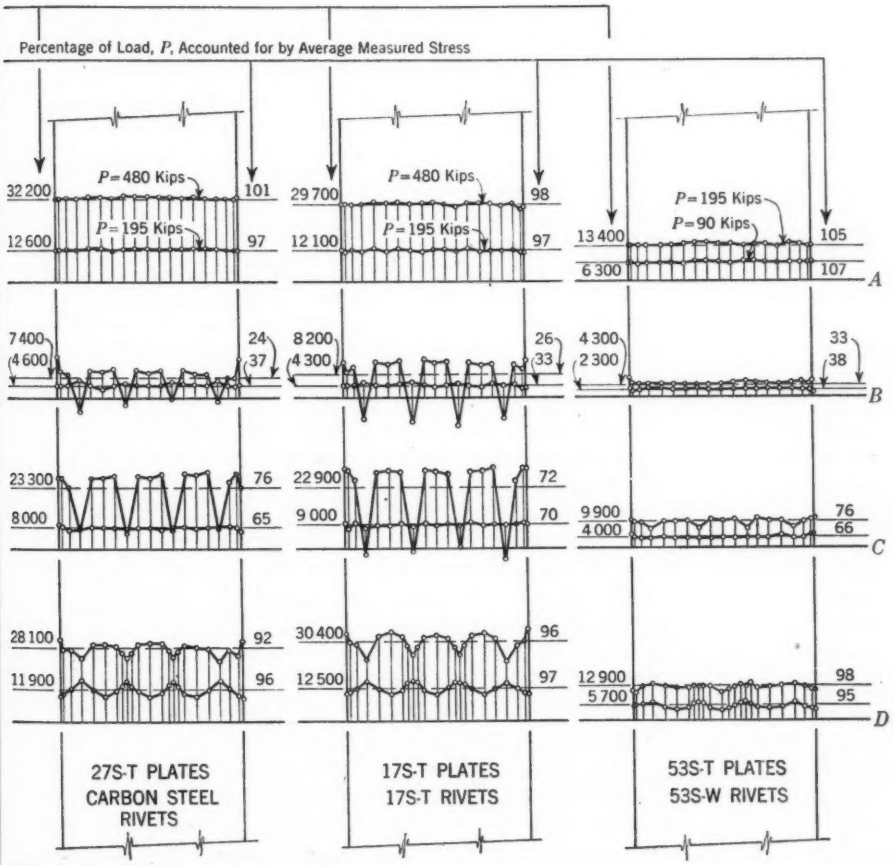


FIG. 8.—MEASURED STRESS DISTRIBUTION IN
(Where Values of *P* Are not

Table 8 gives a summary of the ultimate shear strengths obtained from the auxiliary control specimens. Three tests were made for each combination of material—two on specimens having four rivets and one on a specimen having only two rivets. The distribution of load in all of these joints was presumably known. As will be noted, the results obtained from the joints of the same material were remarkably uniform. Slightly higher strengths were found for

the steel rivets in 27S-T plates than for the same kind of rivets in steel plates, although the difference was too small to appear significant. It is of interest to point out in connection with the shear strengths given for the steel rivets that tests on this material in an Amsler shear tool using a specimen $\frac{1}{2}$ in. in diameter machined from the $\frac{7}{8}$-in. rivet stock indicated an average strength of approximately 44,000 lb per sq in. From this result it appears that the full



WIDE JOINTS DESIGNED TO FAIL IN SHEAR
Given Refer to Curves Above)

shear strength of the material was not obtained in the type of rivet used in the joints. Considerable bending was observed in the rivets in all cases and this behavior was aggravated, of course, by the fact that the cover plates pulled away from the main plates as the ultimate strength was approached. The use of rivets with full-size heads would undoubtedly have resulted in higher shear strengths.

The maximum measured plate slips included in Table 8 give an idea of the deformations produced in the joints before failure occurred. There was no marked difference between the behavior of the steel and aluminum joints, considering the loads involved. These slip determinations seem reasonably consistent with those of Messrs. Davis, Woodruff, and Davis,[6] who reported that shear failures occurred in $\frac{7}{8}$-in. rivets at a detrusion of about 0.3 in.

TABLE 8.—SHEAR STRENGTH OF

No.	Material		Ultimate Load, in Kips			Load per Rivet, in Kips		
	Plates	Rivets						
....	Specimen.....		A	B	C	A	B	C
1	Silicon	Carbon	96.5	97.3	49.0	48.25	48.65	49.0
2	Carbon	Carbon	99.0	98.6	49.6	49.50	49.30	49.6
3	27S-T	Carbon	100.0	99.7	50.8	50.00	49.85	50.8
4	17S-T	17S-T	90.0	89.2	44.7	45.00	44.60	44.7
5	53S-T	53S-W	53.3	52.8	27.5	26.65	26.40	27.5

[a] Readings were taken across the gap between the main

Table 9(*a*) gives a summary of the ultimate loads obtained on the wide joints with the corresponding average shear, tensile, and bearing stresses, maximum measured plate slips and predicted loads. Shear failures were obtained in all cases at loads averaging within about 2% of the predicted values, the latter based on the shear strengths obtained from the auxiliary control specimens. The obvious conclusion to be drawn from this result is that the nonuniform distribution of load observed within the elastic range was not a significant factor as far as ultimate strengths were concerned. Differences in ratios of yield strength to ultimate strength, in elongation values, or in moduli of elasticity, likewise appeared to have no bearing upon the results obtained. Factors of safety on the shear design loads against ultimate failure ranged from 3.0 for the steel rivets to 3.6 for the 17S-T rivets.

In connection with the uniformity of the shear strengths obtained in these tests in which one to twelve rivets were sheared simultaneously, a similar result has been found in tests of hull plate splices.[11] Joints containing both carbon steel and 53S-W rivets, in which nine to sixteen rivets were sheared simultaneously, gave average shearing strengths ranging from 92% to 100% of the strengths obtained for similar rivets tested in auxiliary control specimens. Considerable bending was observed in the rivets and this action naturally resulted in the cover plates pulling away from the main plates. The rivets themselves sheared on only one plane and were pulled through the main plates by the bending in the opposite cover plates. In the steel joints there was the customary scaling of the plates around the rivets, although it should be emphasized

[11] "Static and Repeated Load Tests of Aluminum Alloy and Steel Riveted Hull Plate Splices," by R. L. Templin and E. C. Hartmann, *Technical Paper No. 5*, Aluminum Research Laboratories, 1941.

that this action was not observed until some time after it was evident from the slip measurements that yielding was taking place.

Riveted Joints Designed to Fail in Tension

Fig. 1(*b*) shows the dimensions of the joints included in this series of tests. The joints were similar to those designed for shear (Fig. 1(*a*)), except for the

Auxiliary Control Specimens

Corresponding Average Shear Stress, in Kips per Square Inch				Maximum Measured Plate Slip,[a] in Inches			Load per Rivet, in Kips, Corresponding to Slip		
A	B	C	Average	A	B	C	A	B	C
40.1	40.4	40.7	40.4	0.21	0.21	0.20	45.0	45.0	45.0
41.2	41.0	41.2	41.1	0.29	0.23	0.21	47.5	47.5	45.0
41.6	41.4	42.2	41.7	0.21	0.25	0.24	42.5	45.0	45.0
37.4	37.1	37.2	37.2	0.19	0.19	0.17	42.0	42.0	42.0
22.2	22.0	22.9	22.4	0.18	0.18	0.17	25.0	25.0	25.0

[a] plates at the center of specimens, as indicated in Fig. 7.

width of plates and the transverse spacing of rivets. The principal points of interest are, as before—the distribution of load between the multiple rows of rivets; the load at which first yielding or permanent slip occurred; the measured distribution of strain in the plates; and the ultimate strengths.

Fig. 9 shows the load-slip curves for the three rivet sections investigated in each joint. The characteristics of these curves are similar to those in Fig. 6 and Table 4 for the wide joints designed for shear. The fact that the measured slip for the end rivet sections A and C was greater than that observed for the intermediate section B again emphasizes the difference in the effectiveness of these three rivet rows. As before, the load-slip relationships shown are not linear and there is no definite knee or break in the curves to indicate a point of sudden yielding or slip. It will be noted, however, that the increase in yielding in the steel joints is somewhat more pronounced than in those of aluminum. Although a definite yield point cannot be selected from these curves, the margin between the design loads, indicated on the figures, and the loads producing a considerable increase in slip appeared somewhat greater than was found for the joints designed for shear.

Table 10 gives a summary of the measured slips for sections A, B, and C, and the corresponding load distribution based on the relative magnitudes of the slip observed at the tensile design loads. The percentage of the total load carried by the rivets on each of the end sections A and C ranged from 37% to 42% of the total, and the loads on the intermediate section B ranged from 16% to 21% of the total. The differences between the supposedly like sections A and C are less than those found for the wide shear joints. The results are consistent also in indicating that the rivets in row B are less effective than in the wide shear joints.

Fig. 10 shows the load-slip curves for all joints obtained from measurements taken across the gap between the main plates at the center of the joints. The yielding of the joint composed of carbon steel plates is in marked contrast to

TABLE 9.—STRENGTH OF WIDE-RIVETED AND PIN-CONNECTED JOINTS

Material		Ultimate load (lb)	Stresses (Lb per Sq In.) Corresponding to Ultimate Load			Plate slip (in.)	Load[b] (lb)	Type of failure	Predicted shear load (lb)	Ratio: Col. 10 / Col. 3
Plates	Rivets		Shear	Tension[a]	Bearing					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(*a*) Designed to Fail in Shear; 20¼ In. Wide, Riveted										
Silicon	Carbon	585,400	40,500	46,600	74,200	0.22[c]	540,000	Shear	583,000[d]	0.997
Carbon	Carbon	586,900	40,600	46,100	73,500	0.24[c]	480,000	Shear	596,000[d]	1.015
27S-T	Carbon	585,000	40,500	47,200	75,300	0.20[c]	480,000	Shear	602,000[d]	1.028
17S-T	17S-T	512,000	35,500	41,000	65,400	0.20[c]	480,000	Shear	537,000[d]	1.050
53S-T	53S-W	324,000	22,400	26,300	42,000	0.15[c]	285,000	Shear	323,000[d]	0.998
(*b*) Designed to Fail in Tension; 12½ In. Wide, Riveted										
Silicon	Carbon	563,200	39,000	83,400	71,400	0.32[c]	510,000	Shear	583,000[e]	1.035
Carbon	Carbon	437,200	30,300	64,300	55,100	0.64[c]	437,000	Tension[f]	409,500[e]	0.938
27S-T	Carbon	405,900	28,100	61,000	52,200	0.13[c]	390,000	Tension[f]	389,000[e]	0.958
17S-T	17S-T	417,900	28,900	62,200	53,300	0.12[c]	340,000	Tension[g]	425,000[e]	1.016
53S-T	53S-W	261,900	18,200	39,600	33,900	0.08[c]	220,000	Tension[g]	255,700[e]	0.976
(*c*) Designed to Fail in Bearing; Pin-Connected										
Silicon	Carbon	200,800	31,900	44,200	132,000	0.35[i]	170,000	Shear[h]	....	....
Carbon	Carbon	201,200	32,000	43,900	131,500	0.36[i]	140,000	Bearing	....	....
27S-T	Carbon	153,200	24,400	33,400	100,100	0.16[i]	120,000	Bearing	....	....
17S-T	17S-T	145,600	23,200	32,900	98,500	0.22[i]	120,000	Bearing	....	....
53S-T	53S-W	101,500	16,200	23,000	69,000	0.13[i]	70,000	Bearing	....	....
(*d*) Designed to Fail in Shear; Pin-Connected										
Silicon	Carbon	248,300	39,500	54,600	82,000	0.13[i]	200,000	Shear	....	....
Carbon	Carbon	258,900	41,200	56,600	84,600	0.10[i]	180,000	Shear	....	....
27S-T	Carbon	260,700	41,500	56,900	85,200	0.11[i]	180,000	Shear	....	....
17S-T	17S-T	250,700	40,000	56,800	85,000	0.06[i]	170,000	Shear	....	....
53S-T	53S-W	152,700	21,600	35,300	48,800	0.08[i]	110,000	Shear	....	....

[a] Based on the net areas given in Table 1. [b] The load corresponding to the plate slip given in Col. 7. [c] Readings for maximum measured plate slip were taken across the gap between the main plates, at the center of the joints, as indicated in Fig. 7. [d] Based on the average shear strengths obtained from the auxiliary specimens given in Table 8. [e] Based on the tensile strengths of parallel specimens given in Table 2 and shear strengths in Table 8. [f] ¾-in. plate. [g] ⅝-in. plates. [h] Shear and bending of pin. [i] Readings for maximum measured plate slip were taken diagonally across the main plates and cover plates, at sections through the pins.

that shown for the other materials. Fig. 11 shows the load-strain curves obtained from the measurements of combined strain and slip on the 48-in. gage lengths. Since these curves are more indicative of strain in the plates than slip between the plates, a fairly definite elastic range was observed. The total measured elongations within the elastic range were from 9% to 12% less than computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500 and 10,300 kips per sq in. for steel and aluminum, respectively.

Fig. 12 shows the measured distribution of longitudinal stress in all the joints of this series for two increments of load and gives the percentages of total applied load accounted for by the average measured stress on each of the

TABLE 10.—MEASURED PLATE SLIP AND CORRESPONDING LOAD DISTRIBUTION IN RIVETED JOINTS 12½ IN. WIDE, DESIGNED TO FAIL IN TENSION

MATERIAL		Tensile design stress (lb per sq in.)	Corresponding design load[a] (lb)	MEASURED SLIP[b] (IN.)			PERCENTAGE OF TOTAL LOAD CARRIED BY EACH SECTION[c]		
Plates	Rivets			Section A	Section B	Section C	Section A	Section B	Section C
Silicon	Carbon	24,000	162,000	0.0015	0.0007	0.0014	42	19	39
Carbon	Carbon	18,000	122,000	0.0010	0.0005	0.0009	42	21	37
27S-T	Carbon	21,000	140,000	0.0025	0.0010	0.0025	42	16	42
17S-T	17S-T	16,000	107,500	0.0026	0.0013	0.0027	39	20	41
53S-T	53S-W	12,000	79,400	0.0020	0.0010	0.0022	39	19	42

[a] Based on net areas given in Table 1. [b] Determined from load-slip curves shown in Fig. 9. [c] Determined from relative magnitude of measured slips on sections A, B, and C.

sections investigated. Similarly to Fig. 8, the values indicated are the average measured unit strains multiplied by the moduli of elasticity (29,500 and 10,300 kips per sq in., respectively). Values of load P for sections B, C, and D are the same as for section A. On section A in the main plates the distribution was reasonably uniform (as would be expected) and the average values of stress were within a few per cent of those computed. On sections B, C, and D, however, the distribution was quite varied. The percentages of load accounted for by the average measured stresses on sections B and C are not as consistent, nor can they be considered as reliable, as the distribution of load obtained from the slip measurements. On section D, the average measured stresses were, in most cases, in fair agreement with the corresponding computed values. Perhaps the most significant point to be emphasized is that the strain measurements indicated essentially the same characteristics of behavior for all combinations of materials.

Table 9(*b*) gives the ultimate strengths for all the joints of this series, with the corresponding shear, tensile, and bearing stresses, maximum measured slips, and predicted loads. Tensile fractures, either in the main plates or the cover plates, were obtained in all joints except the one composed of silicon steel plates and carbon steel rivets, where a shear failure was obtained. The latter result was expected, however, after the results of the auxiliary shear tests indicated rivet strengths somewhat less than those relied upon in the original design. The predicted shear load was within about 3% of that actually obtained. The predicted tensile loads, based on the properties of the plate material given in Table 2, were within 2% to 6% of the actual values.

Of the four cases of tensile fractures listed in Table 9(*b*), three show plate tensile strengths which were higher than the tensile strengths of the material determined on the control specimens. Recent tests by other investigators[6,10,12,13]

[12] "Design Specifications for Bridges and Structures of Aluminum Alloy 27S-T," prepared by Leon S. Moisseiff for Aluminum Co. of America (1940), Article 205.

[13] "Specifications for Steel Railway Bridges," A.R.E.A. (1938), Article 409.

show this same trend for both silicon steel and carbon steel riveted joints. In comparing the results of these tests with those reported by the other investigators, however, an important difference in the procedure for computing strengths should be noted. In the investigations by Wilbur M. Wilson, M. Am. Soc. C. E., and F. P. Thomas,[10] and by Messrs. Davis, Woodruff, and Davis,[6] the net section areas were based on hole diameters arbitrarily taken as $\frac{1}{8}$ in. greater than the nominal rivet diameters, a procedure commonly required by

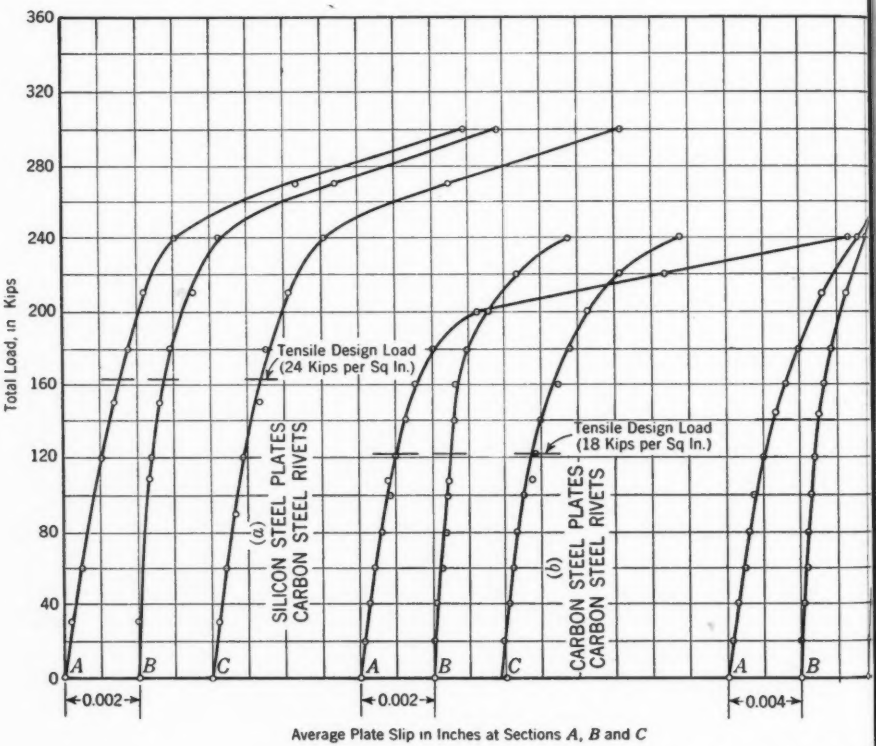
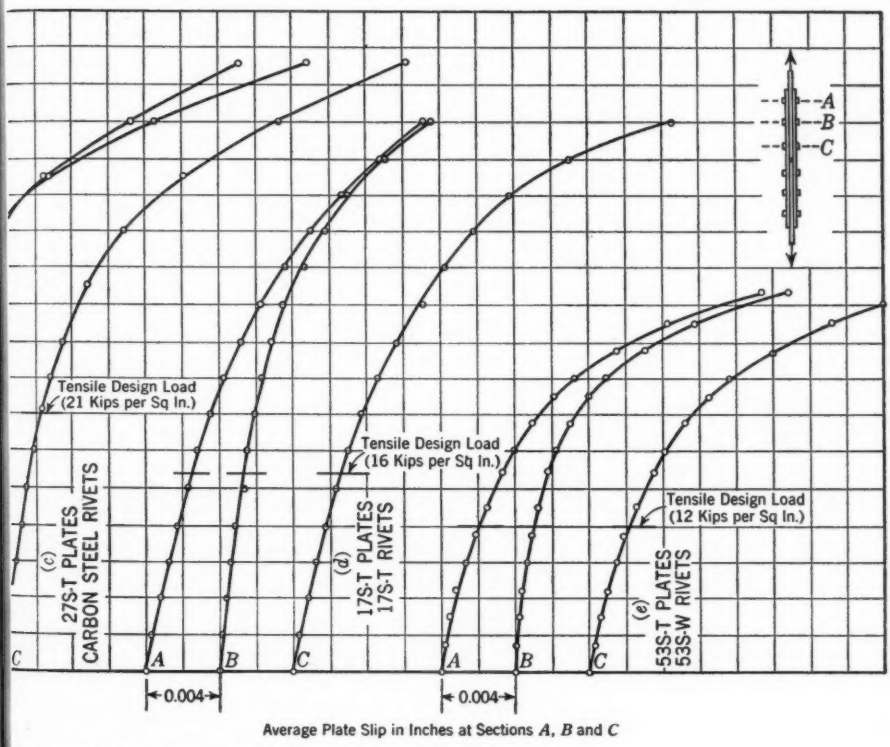


FIG. 9.—LOAD-SLIP CURVES FOR NARROW JOINTS DESIGNED TO FAIL I

design specifications.[12] In these tests, however, the net section areas were based on the nominal rivet hole diameters, a procedure believed to be entirely justified for investigational purposes. Had the other procedure been used in these tests, the tensile strengths on the net sections would have been increased about 6% more than the values given in Table 9(*b*). Although it has not been possible to show a correlation between the tensile design loads and the loads producing the first appreciable permanent slip, the factors of safety against ultimate tensile failure ranged from 2.9 for the 27S-T joint to 3.9 for the 17S-T joint.

Fig. 13 shows typical examples of the tensile fractures obtained. In the case of the joint composed of carbon steel plates, the slip lines, elongation of the rivet holes, and lateral contraction of both main and cover plates at the most highly stressed rivet sections are clearly indicated. The first evidence of slip lines on the surface of the plates in this joint was obtained at a load of approximately 240 kips, or only a little more than one half the breaking load. The gap opening between the main plates at the center of the joint (originally



TENSION (12½ IN. WIDE, WITH 24, ¾-IN. RIVETS; 10-IN. STRAIN GAGE)

closed) was 0.64 in., and this evidence of ductility is clearly indicated in the photograph.

Fig. 13(*b*) shows that failure in the 27S-T joint was accompanied by almost no elongation of the rivet holes, lateral contraction of the plates, or opening of the gap at the center of the joint. The opening between the main plates, determined after fracture, was only 0.08 in., as compared to the opening of 0.64 in. observed for the carbon steel joint. Despite this apparent lack of ductility it should be emphasized that the full strength of the 27S-T plate material was developed. Fig. 13(*c*), which shows the failure of the 17S-T joint, indicates a behavior more like that obtained for the carbon steel joint.

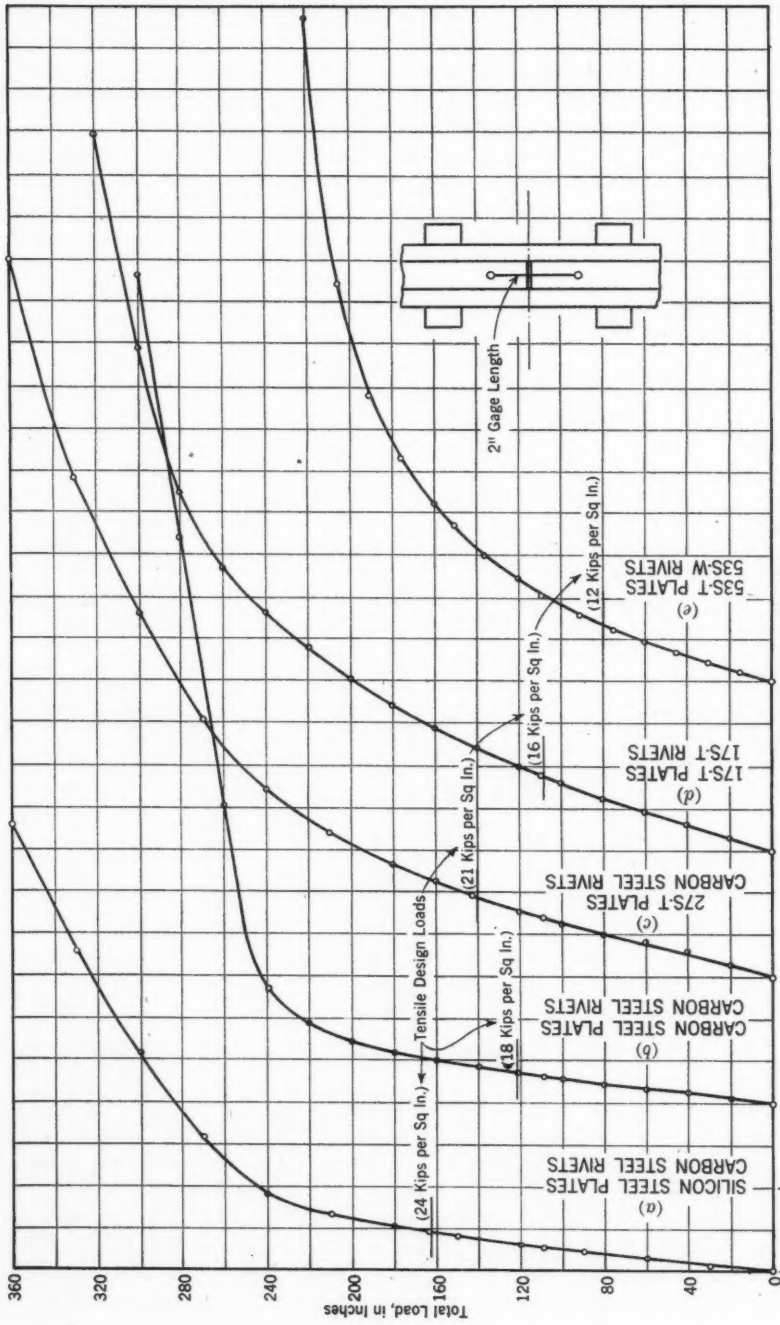


FIG. 10.—LOAD-SLIP CURVES FOR NARROW JOINTS DESIGNED TO FAIL IN TENSION; MEASUREMENTS TAKEN ACROSS THE GAP BETWEEN THE MAIN PLATES AT THE CENTER OF THE JOINTS; 2-IN. GAGE LENGTHS (12½-IN. WIDE, WITH 24, ¾-IN. RIVETS)

THE CENTER OF THE JOINTS; 2-IN. GAGE LENGTHS (12½-IN. WIDE, WITH 24, ¾-IN. RIVETS)

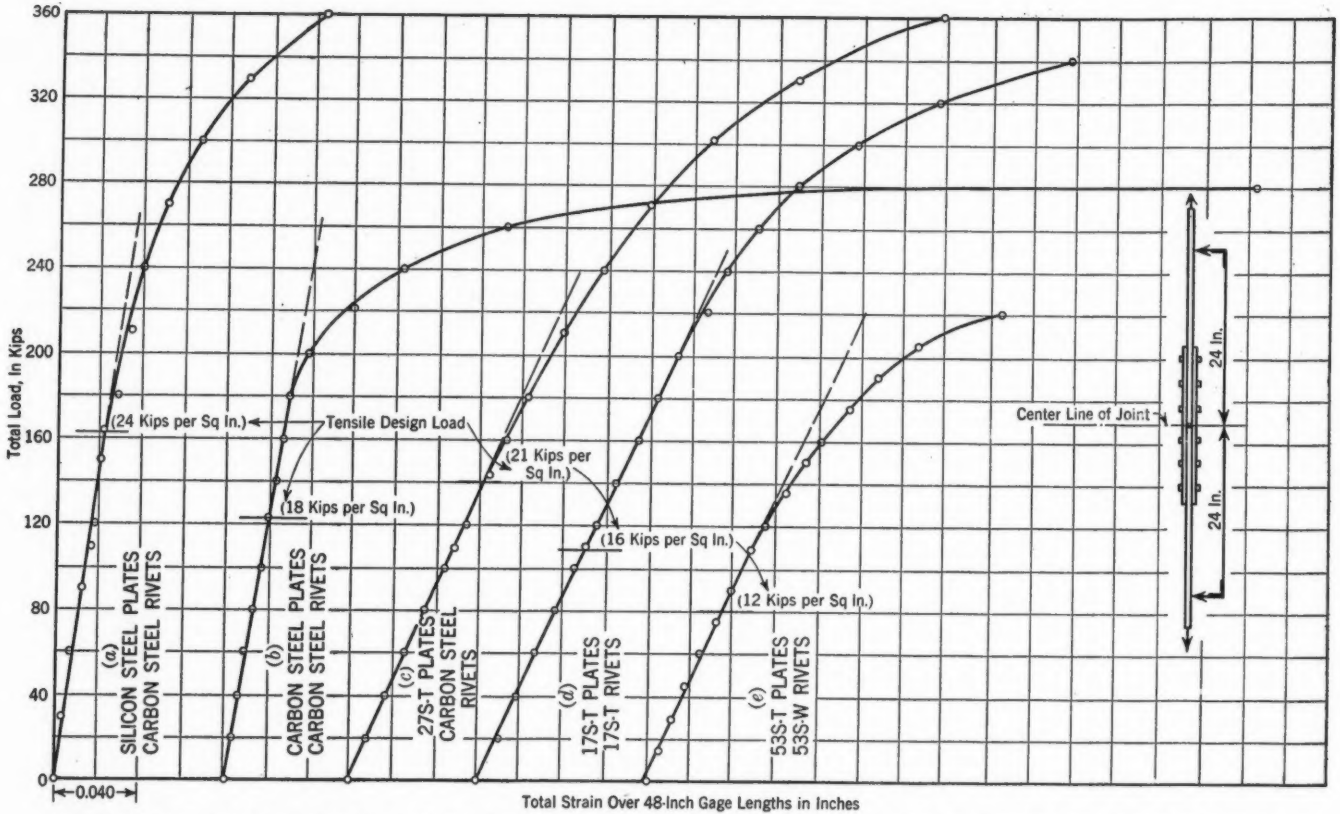


FIG. 11.—LOAD-STRAIN CURVES FOR NARROW JOINTS DESIGNED TO FAIL IN TENSION; STRAIN MEASURED BY DIAL INDICATOR ON 48-IN. GAGE LENGTHS (12½-IN. WIDE, WITH 24, ¾-IN. RIVETS)

Pin-Connected Joints Designed to Fail in Bearing

Fig. 2(*a*) gives the dimensions of the joints included in this series. All strain measurements were made with the nuts and washers removed from the pins, to permit access to the strain gage lines. At the conclusion of these measurements the nuts were replaced and the tests carried to failure.

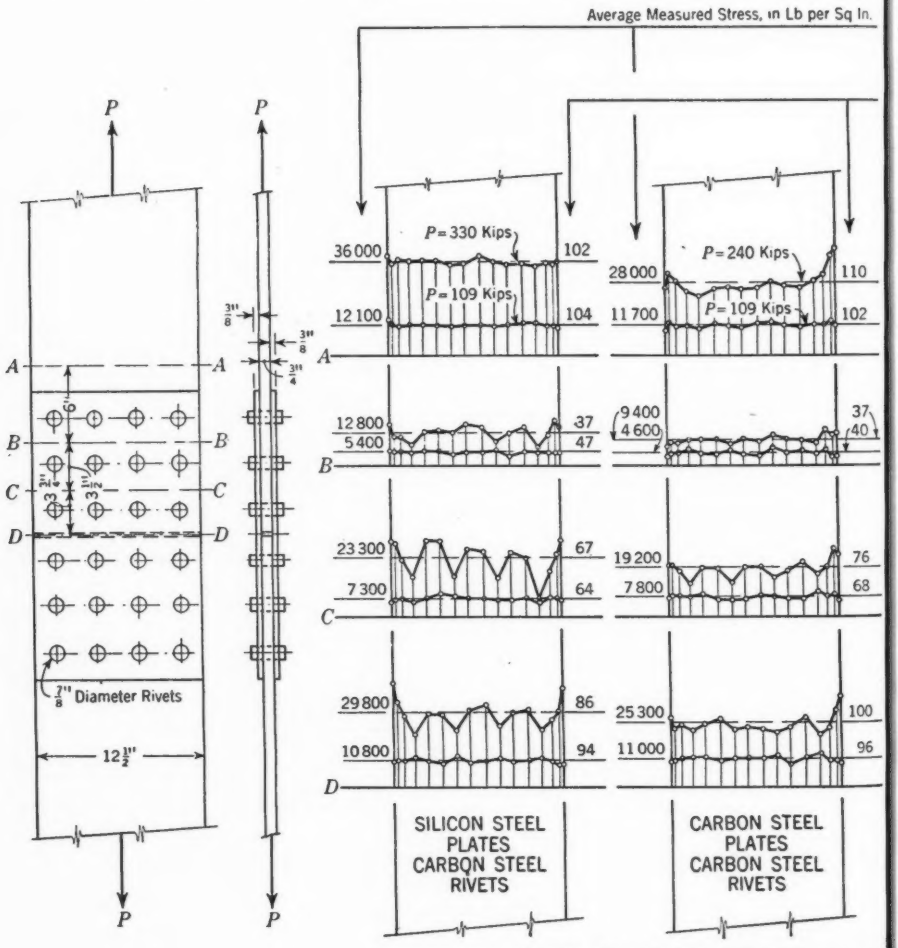
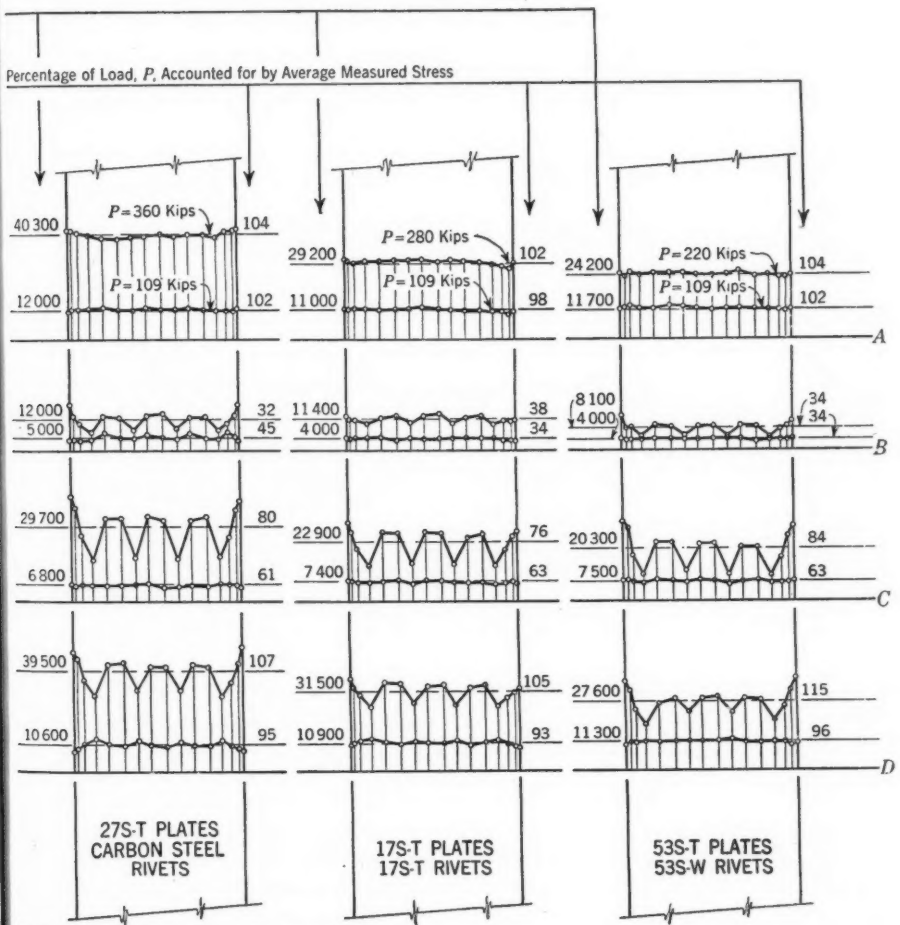


Fig. 12.—Measured Stress Distribution in
(Where Values of *P* Are not

As was the case in all the previous tests described, the relationships observed between load and slip were not linear and there was no definite point of first yielding. The margin between the loads corresponding to the bearing design stresses for the plate materials and the loads for which appreciable yielding was found appeared to be somewhat greater than that shown in Fig. 6

(or by the data in Table 4) for the wide joints designed for shear. Load-slip curves for the pin-connected joints have not been included herein because of their similarity to the numerous curves of this type already shown.

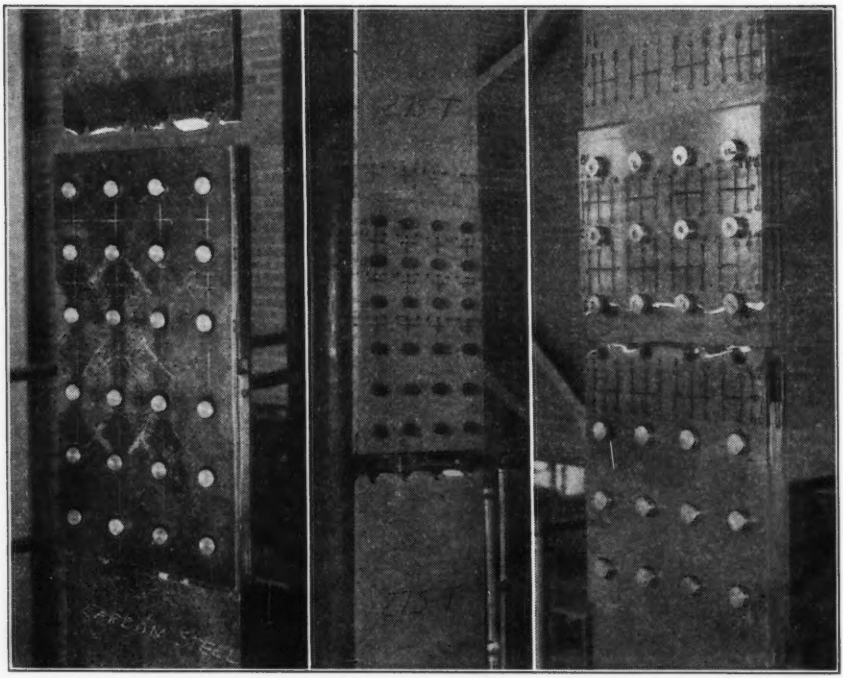
Fig. 14 shows the measured distribution of stress in these joints. Again (see Fig. 8), the values indicated are average measured unit strains multiplied



Narrow Joints Designed to Fail in Tension
Given Refer to Curves Above)

by the moduli of elasticity of steel and aluminum (29,500 and 10,300 kips per sq in., respectively). As may be noted, essentially the same behavior was observed for all materials. On section A, which was considerably removed from the influence of the pin, a reasonably uniform distribution of stress was observed and the average measured values were within a few per cent of those

computed. On section B the effect of the pin was more pronounced although, in most cases, the averages of the measured stresses were within a few per cent of those computed. On section C, through the center of the pins, the range of



(a) Carbon Steel Plates and Rivets (b) 27S-T Plates and Carbon Steel Rivets (c) 17S-T Plates and Rivets

FIG. 13.—TENSILE FRACTURE OF A NARROW JOINT (12½-IN. WIDE)

stresses observed is clearly indicative of the high stress concentrations which must have been present at the boundaries of the pin openings. In view of the limited scope of the strain measurements and the fact that the strains on the inside gage lines exceeded the elastic limit for relatively low loads, no attempt has been made to make any comparisons between average measured stresses and the corresponding computed values. According to previous tests and photoelastic measurements[14] the stress concentration factor at the edge of a pin opening in plates of the proportions used should be approximately 4.7. Although it has not been possible to obtain a check on this concentration factor, it is of interest to point out that if such a stress concentration is assumed, the loads computed for first local yielding would correspond approximately to the design values for bearing. These computed loads were in every case below

[14] "Stress-Concentration Factors Around a Central Circular Hole in a Plate Loaded through Pin in the Hole," by M. M. Frocht and H. N. Hill, *Journal of Applied Mechanics, Transactions*, A.S.M.E., March, 1940, pp. A5–A9.

the points on the load-slip curves at which some permanent yielding seemed to have occurred.

The strain measurements taken on section D, Table 11, represent the only attempt to measure bearing stresses directly. Even though it was obviously

TABLE 11.—MEASURED STRESS DISTRIBUTION AT SECTION D (FIG. 14) IN JOINTS DESIGNED TO FAIL BY BEARING
(Stresses Are in Kips per Square Inch)

Description	Plate Material									
	Silicon steel		Carbon steel		27S-T		17S-T		53S-T	
Material in pins	Carbon steel		Carbon steel		Carbon steel		17S-T		53S-W	
Loads, in kips	44	110	44	85	44	85	44	85	44	63
Measured Stresses:[a]										
Longitudinal	−4.4	−41.6	−3.4	−38.4	−6.6	−20.7	−8.2	−48.9	−11.9	−45.1
Transverse	+4.4	+13.0	+3.8	+ 5.3	+3.4	+ 6.8	+4.3	+ 2.0	+ 3.0	− 1.3

[a] Based on longitudinal and transverse strains assuming elastic action.

not possible to obtain maximum values from measurements over 2-in. gage lengths, centered 1½ in. from the edge of the pins, the stresses corresponding to the strains measured were in all cases greater than the maximum values found on section C for the same loads. Table 11 gives the longitudinal and transverse stresses for some of the same loads for which the stress distribution on section C was shown in Fig. 14. Values for the maximum loads have not been included because the corresponding strains were outside the range of the measuring instruments used. Since there was some tendency for bending in the ends of the cover plates this action may have been partly responsible for the large strains observed.

Table 9(*c*) gives a summary of the ultimate loads developed by these single-pin joints, with the corresponding shear, tensile, and bearing stresses and the maximum measured slips. So-called bearing failures were obtained for all but the joint composed of silicon steel plates, in which case the pin was badly bent and finally sheared off at a stress considerably below the nominal shear value for steel.

Fig. 15 shows the different types of failures obtained. A typical bearing failure involved a combination of elongation of the hole, bending of the pin, and buckling of the free ends of the cover plates. The ultimate bearing stresses obtained from such a test are so dependent upon the relative proportions of the plates and pin and the amount of lateral restraint provided against the bulging of the plates that they can scarcely be considered as representing definite properties of the materials. It may be pointed out, however, that the ultimate bearing strengths obtained for the silicon and carbon steels were approximately 1.4 and 2.1 times the tensile strengths, whereas the corresponding ratios for the aluminum alloys ranged from about 1.6 to 1.8. The factors of

safety on the bearing design stresses used ranged from a minimum of 3.3 for the 27S-T to 4.9 for the carbon steel.

Pin-Connected Joints Designed to Fail in Shear

Fig. 2(*b*) shows the dimensions of this series of joints. The tensile and shear areas were nominally the same as in the single-pin joints designed to fail

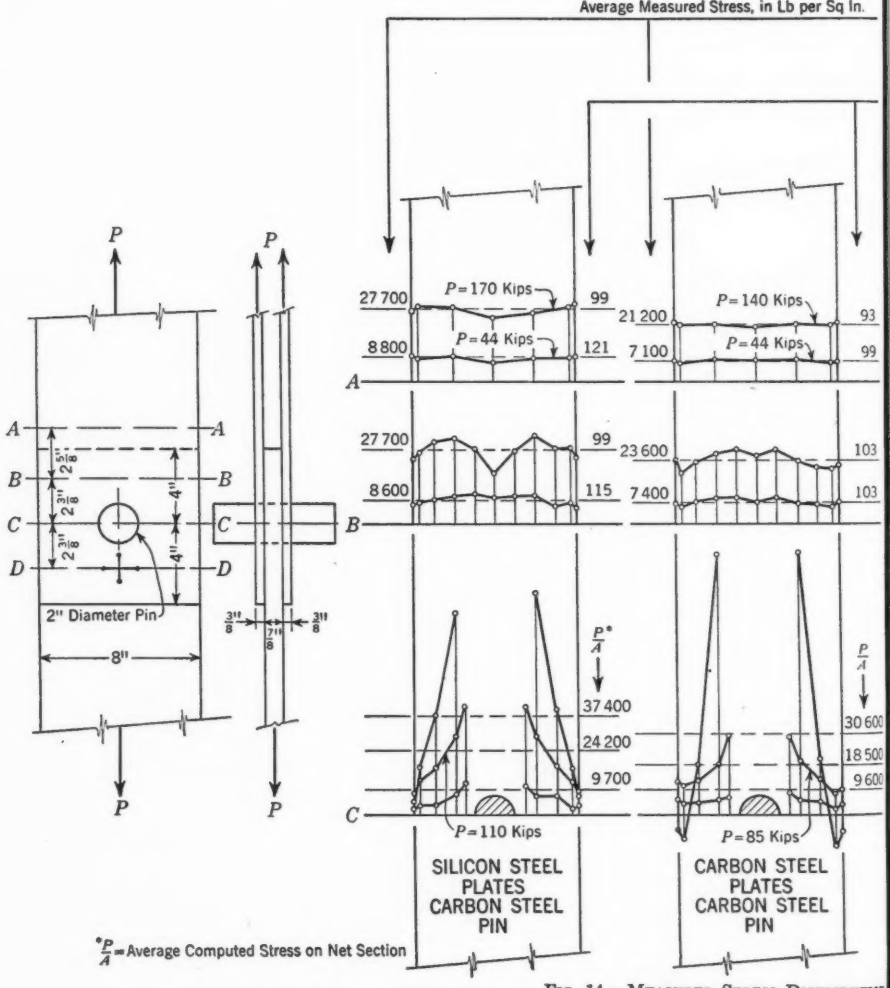
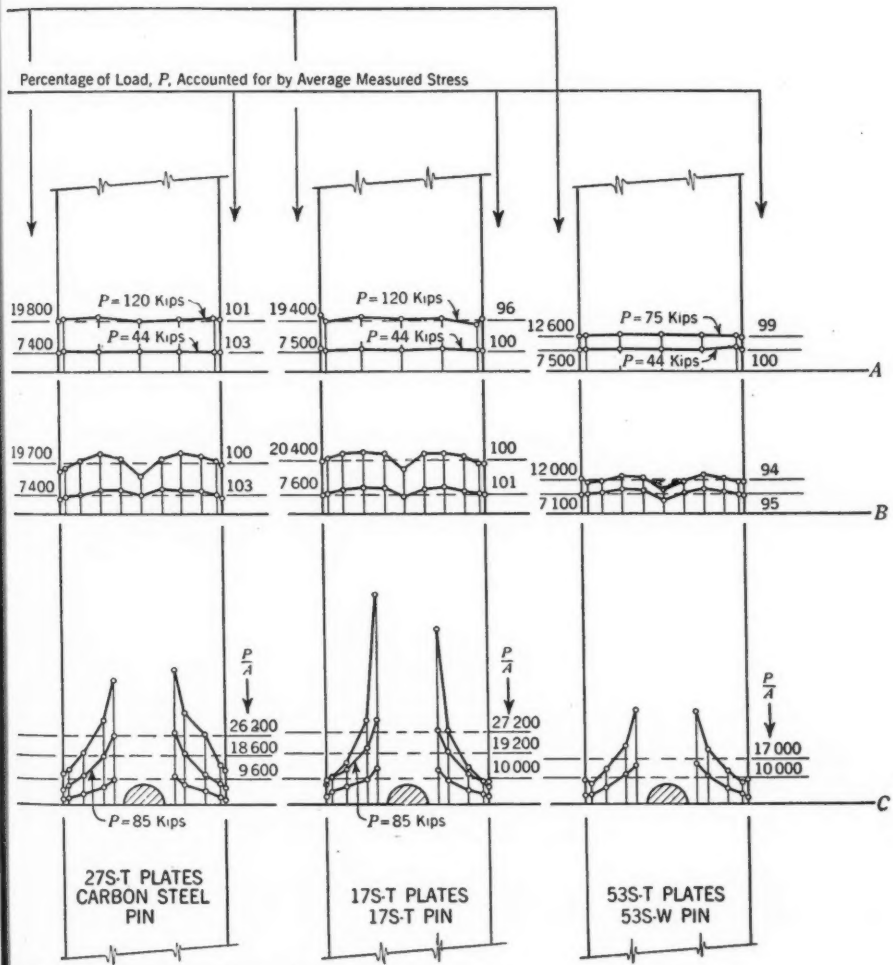


Fig. 14.—Measured Stress Distribution (Where Values of *P* Are not

in bearing but the bearing areas were 100% greater. The first loadings were made with the nuts and washers removed, but these were replaced before completing the tests to failure.

Since the load-slip curves for these joints exhibited the same general characteristics of behavior as were found for the other types of joints, none of them have been included here. The yielding in the joints with the steel pins was somewhat more pronounced than in the other cases, but the margin between the shear design loads and this yielding was about the same. The measured slips indicated an equal distribution of load between the two rows of pins. As



IN JOINTS DESIGNED TO FAIL BY BEARING
Given Refer to Curves Above)

was the case for all the different types of joints previously considered, essentially the same behavior was found for all combinations of material. On the sections outside the influence of the pins, the distribution of stress was reason-

ably uniform, particularly for the smaller loads, and the average measured stresses were in fair agreement with the corresponding computed values. On the sections midway between pins, the distribution of stress was extremely varied and the stress picture obtained was not complete enough to give a very satisfactory comparison of average measured and computed values.

Table 9(*d*) gives a summary of the ultimate loads with the corresponding shear, tensile, and bearing stresses and maximum measured slips. As may be noted from a comparison with the results given in Tables 8 and 9(*a*), the pins developed essentially the same shear strengths as the $\frac{7}{8}$-in. rivets used. The greatest difference was an increase in shear strength of about 8% found for the 17S-T pins. The pins were all slightly bent as well as sheared and the holes in the cover plates were somewhat elongated.

Summary

1. The load-slip curves for all types of joints and for all the different combinations of materials were characterized by the following features:

(*a*) Slip occurred with the first application of load, which indicated no appreciable friction between the plates.

(*b*) The relation between load and slip was not linear, although the permanent slips measured for loads equivalent to the design values were on the order of only a few ten thousandths of an inch.

(*c*) There was no definite yield point on any of the curves, although the yielding of the steel joints was more pronounced than that of the aluminum joints.

2. The load-slip curves obtained for individual sections in all joints having two rows of rivets or pins indicated an equal distribution of load between rows, as would be expected.

3. In the auxiliary control specimens the ratios of load-to-slip corresponding to the shear design loads for the aluminum plates and rivets were approximately one third those obtained for the all-steel joints, which is consistent with the ratio of the moduli of elasticity of these materials. The load-slip ratio for the aluminum alloy, 27S-T, joint with steel rivets was 43% of that found for the all-steel joints, indicating the importance of plate action upon the amount of slip observed.

4. The load-slip curves obtained for the large joints having three rows of rivets in each half indicated that within the elastic range the end rows carried more load than the intermediate rows. From the relative magnitudes of the slips measured, it appears that the end rows each carried from 33% to 43% of the total load, whereas the intermediate rows carried from 16% to 26% of the total. These variations from a uniform distribution of load are greater than indicated by the elastic theory.

5. Measurements of combined slip and strain made on the large joints over 48-in. gage lengths indicated total elongations approximately 10% less than computed for the same lengths of unspliced main plate, assuming moduli of elasticity of 29,500 and 10,300 kips per sq in. for the steel and aluminum alloys, respectively.

6. The measured longitudinal strains agreed in general with the type of behavior expected on the sections investigated. The interpretation of these strains on the basis of a unidirectional state of stress gave average stresses which, in most cases, were in reasonably good agreement with the averages computed.

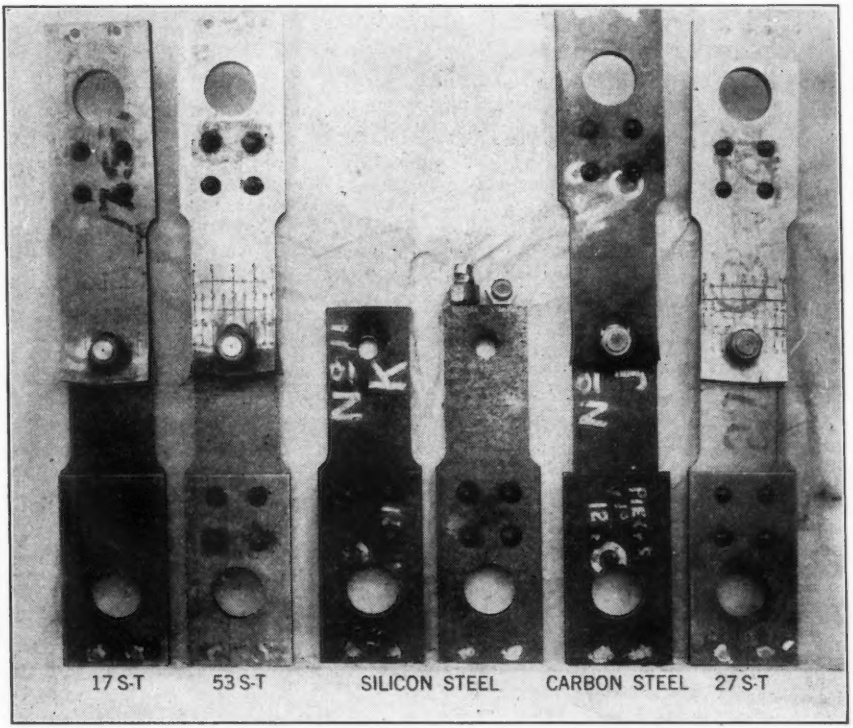


FIG. 15.—FAILURES IN NARROW JOINTS; 8 IN. WIDE, WITH A PIN 2 IN. IN DIAMETER

7. The maximum variations from a uniform distribution of measured stress in the riveted joints occurred in the cover plates on the gage lines in line with the rivets. For average tensile stresses well within the yield strength of the material, the individual stresses on these gage lines were in many cases found to be in compression.

8. The strains measured on the sections through the center of the 2-in. pins were clearly indicative of the stress concentrations which must have occurred at the edges of the pin holes, although no quantitative measure of the magnitude of these stress concentrations was obtained.

9. The loads at which the maximum stresses at the hole boundaries for the 2-in. pins were computed to be equal to the yield strength of the material, assuming a stress concentration factor of 4.7 as indicated by other tests, were below those at which first yielding was apparent from the load-slip curves.

10. The ultimate shear strengths obtained for the large joints having twenty-four rivets averaged within about 2% of the values predicted from the

tests on the auxiliary control specimens, having two and four rivets. From this result it appears that the inequality of the load distribution found within the elastic range had no bearing upon the ultimate shear strengths. Differences in elongations, moduli of elasticity, and ratios of yield to ultimate strength likewise appeared to have no bearing upon the strengths of the joints. The factors of safety on the shearing design stresses ranged from 3.0 for the carbon steel rivets to 3.6 for those of aluminum alloy 17S-T.

11. The ultimate strengths of the large riveted joints which failed by tension in the plates averaged within about 4% of the values predicted from the tensile properties of the plate material, assuming net section areas based on nominal rivet hole diameters. The factors of safety on the tensile design stresses ranged from 2.9 for the aluminum alloy 27S-T plates to 3.9 for those of aluminum alloy 17S-T.

12. The ultimate bearing strengths obtained for the silicon and carbon steel plates from the joints having a single 2-in. pin were 1.4 and 2.1 times the tensile strengths, respectively, whereas the corresponding ratios for the aluminum alloys ranged from about 1.6 to 1.8. The factors of safety on the bearing design stresses ranged from 3.3 for the aluminum alloy 27S-T plates to 4.9 for those of carbon steel.

13. The shearing strengths obtained from the joints having four 1-in. or $1\frac{1}{16}$-in. pins were in good agreement with those obtained from the $\frac{7}{8}$-in. rivets. The factors of safety on the shearing design stresses ranged from 2.9 for the carbon steel pins to 4.0 for those of 17S-T.

Conclusions

The most significant conclusion to be derived from the results of these tests is that the behavior of all the large riveted and pin-connected joints was entirely consistent with the basic assumptions generally made in design. Although inequalities in load distribution between multiple rivet rows were clearly indicated within the elastic range, this distribution apparently had no bearing upon static ultimate strengths, whether failures occurred by shear in the rivets or by tension in the plates. The amount of plastic yielding necessary to level off high stress concentrations and to effect a uniform distribution of load among the rivets must be relatively small. Essentially the same behavior was obtained for all the different combinations of the materials investigated. Elongations, moduli of elasticity, and ratios of yield to ultimate strengths were apparently not significant factors as far as the static strengths of these joints were concerned.

Acknowledgment

This paper was prepared under the auspices of the Committee of the Structural Division on Design in Lightweight Structural Alloys. The tests were made in 1939 at the Aluminum Research Laboratories, Aluminum Company of America, in New Kensington, Pa.

The tests described were made under the direction of Mr. Templin, chief engineer of tests, Aluminum Company of America. The writers wish to acknowledge the assistance of Mr. Templin and various members of his staff in planning and conducting these tests.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE

A SYMPOSIUM

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NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 1, 1943.**

FOREWORD

At a technical meeting of the Sanitary Engineering Division on January 22, 1942, with W. Rudolfs, M. Am. Soc. C. E., presiding, the subject of dewatering, drying, incineration, and use of sewage sludge was thoroughly explored. Five of these papers have been revised for publication in the present Symposium. The most important revision is in the isolation of a bibliography on the subject as an independent unit of the Symposium, subject to discussion on a par with any of the papers. Additions to this bibliography that occur in discussion will be appended to the bibliography in the final printing.

DEVELOPMENTS INDICATED BY EXPERIENCE AT BUFFALO, N. Y.

BY C. R. VELZY,[1] M. AM. SOC. C. E.

SYNOPSIS

Sludge from the primary treatment plant at Buffalo, N. Y., passes through tanks for a short period of digestion, and is then disposed of by vacuum filtration and incineration. There have been developments in sludge conditioning and in the operation and maintenance of filters. The major problem with incineration has been severe abrasion, particularly in the flash driers, induced draft fans, and cyclones. Resulting costs of maintenance have been reduced by changes in equipment and materials. Operating results and costs are stated briefly.

The sludge disposal facilities at Buffalo are part of a treatment plant designed for a population of 750,000. The plant provides primary treatment with chlorination. The raw sludge is the product, therefore, of primary sedimentation only. The design contemplated a suspended solids content in the raw sewage amounting to 0.30 lb per capita, and a removal of 50%, or 0.15 lb per capita. For the present population of approximately 600,000, this would result in a daily removal of raw solids of approximately 90,000 lb. For the year 1940–1941 the actual amount of dry solids removed from the sewage was 89,300 lb per day, and was somewhat less than that for the previous two years.

Sludge disposal at Buffalo includes short-period or partial digestion, vacuum filtration, and incineration. Sludge is conditioned for filtration with lime and ferric chloride. After filtration the sludge cake passes over a system of conveyer belts, where it is weighed on its way to drying equipment. The dried and powdered sludge is blown into the furnace and burns in suspension. Hot gases from combustion are used to dry the sludge, and all vapors from drying are returned to the furnaces to eliminate odors.

The raw sludge is pumped to four sludge tanks having a total volume of 750,000 cu ft, or a design capacity of 1.0 cu ft per capita. Each tank is 90 ft in diameter, with a side-wall depth of 27.5 ft, and is equipped with a floating cover.

In the sludge disposal building there are three vacuum filters of 500 sq ft of filtering area each and three sludge incinerators having a design capacity of 100 tons of sludge cake per day on the basis of a moisture content of 70%. The furnace units include cyclones for the separation of vapors from powdered sludge and for the removal of fly ash from the flue gases, a pneumatic ash-handling system, preheaters for the recovery of heat, and the fans and blowers for the handling of gases and powdered sludge. The furnaces are equipped with combination oil and gas burners for the use of auxiliary fuel to supply

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heat to make up deficiencies of heat units in the sludge. During three years of operation this equipment has performed well. Some changes in details have been made, others are under study, and it is expected that more developments will be indicated as the art progresses.

The value of sludge tanks has been shown by the writer elsewhere (1).[2] They absorb the irregularities in quantity of raw sludge obtained, permit a six-day operating week in dewatering and incineration, provide gas as fuel for boilers and auxiliary heat for incinerators, and deliver a sludge of reasonably uniform characteristics for filtration and burning.

In the conditioning of sludge, the laboratory has made determinations of the theoretical quantity of chemicals required and has proposed a procedure by which the operator in the plant can make a quick determination of the amount of chemicals required. In connection with the application of chemicals, experiments have been made on details of the mixing tank with particular reference to baffling, mixing-paddle speeds, and detention. A short detention period with no baffles and proper mixing-paddle speeds is indicated, but more work is to be done before definite conclusions are drawn as to the best combination of these factors. In the equipment for the application of chemicals, there has been a tendency away from automatic devices and toward devices that will provide easy manual control and permit the operator to observe rates of application of chemicals and readily make necessary adjustments. For example, ferric chloride solution is now fed through a perforated rubber pipe, arranged so that the number of small streams of the solution is an indicator of the rate of feed. Thus the operator, or the superintendent, can quickly check the application of ferric chloride by counting the streams on this feeder and observing the dial on the weightometer.

In the operation of vacuum filters, studies have included experiments with different types of filter cloth and a number of modifications in the mechanical operation of the filters. Canton flannel has been found to be most satisfactory in operating results and in general economy in regard to cost and life of cloth. In the application of the cloth to the filter drum, it has been found that ordinary sash cord is a satisfactory substitute for the bronze strips which were originally used. The sash cord is inexpensive, is easy to apply, and results in greatly reduced time for applying new cloth.

On one of the filters a screw feeding device has been installed, with a surge bin above the screw. This device receives the sludge cake from the filter and discharges it uniformly to the first conveyer belt, producing a more satisfactory reading on the weightometer chart. With a variable speed drive, it also provides for a closer control of the loading on the incinerators. The device, as now used on one of the filters, is satisfactory, but a definite determination has not been made as to whether or not it is worth the cost.

After about two years of operation, it was found that the wire screen on the surface of the filter drum had become coated with lime scale so that the areas between the wires were materially reduced. This scale was removed successfully by the application of muriatic acid in the filter tub. Acid also

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography at the end of the Symposium.

has been used successfully in reducing the scale accumulations in filtrate lines and in the supernatant line which receives the discharge from the filtrate pumps.

Fig. 1 shows a flow diagram of the drying and incineration processes. The fundamental principles of these processes have been described by D. C. Reybold (2). This flow diagram represents the present installation at Buffalo and differs in some mechanical details from the original installation. It also differs from other installations previously described. This discussion of problems and improvements is offered, not as a criticism of the original design, but as a description of developments normal to any process as new as sludge incineration.

The developments actually accomplished thus far are largely in the mechanical operation of the drying and burning equipment. The major problem has been due to abrasion of certain parts of the equipment. It appears that this could not be foreseen in the original design, as in other installations abrasion has been much less severe. In fact, it has been stated that in one of the Chicago, Ill., installations certain parts, which suffer from abrasion at Buffalo, give trouble because of the formation of scale. This is probably due to differences in the characteristics of sludge and perhaps in the character of auxiliary fuel.

In this connection, it is interesting to note that, during the first year of operation at Buffalo, when oil was used as an auxiliary fuel, scale formed on the preheater surfaces to such an extent that cleaning was required at frequent intervals. Since gas has been used exclusively as auxiliary fuel, the scale on the preheaters has disappeared, the soot blowers have become unnecessary and have been removed, and abrasion on the heater surfaces has made it necessary to replace some of the parts.

Abrasion has been particularly severe on the flash dryer, the induced draft fan, and at bends in pipe lines and ducts handling gases, powdered sludge, and fly ash. In the pipe lines and ducts this problem has been solved rather easily by the use of concrete wear backs at the bends. The concrete can be used in a thickness which will withstand wear for a considerable time and is easily replaced when it is worn through.

Wear on the flash dryer during the first months of operation was so severe on the bars and rings that the cages required replacement after about two months of continuous operation. At the suggestion of the manufacturer, a lower speed was tried in the cage mill. This has increased the life of the mill to about five months. It is possible that further improvement can be made. Adjustments in the type of assembly have resulted in easy replacement of worn parts and the reuse of portions of these parts.

The wear from abrasion on the induced draft fan has been so severe that originally the fan blades would last only approximately two months. The repair was difficult and costly because it required disassembling of the fan, replacement of blades, and reassembling of the fan. Hard surfacing of blades by arc welding was tried, but the slight additional wear did not warrant the cost. Ribbing of the blades with hard material was as effective as complete surfacing, and much less costly. The real answer was found in this case when

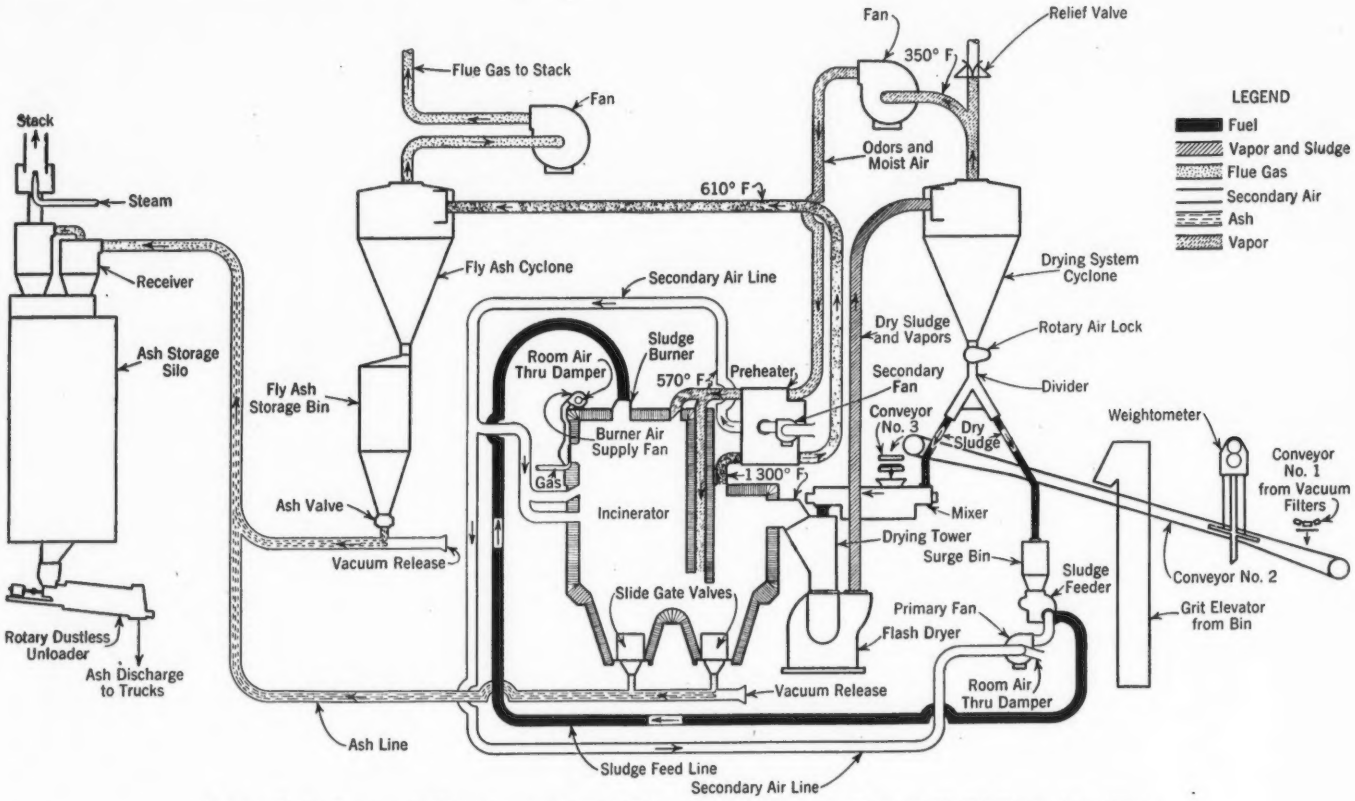

FIG. 1.—FLASH DRYING AND INCINERATION SYSTEM; TREATMENT WORKS OF THE BUFFALO (N. Y.) SEWER AUTHORITY

the fan was moved to the clean air side of the cyclone. One unit in this location has operated for more than a year to date (1943) without showing appreciable wear.

In the fly ash, cyclone abrasion has been less severe; nevertheless, it has constituted a problem calling for careful attention. Gunite has been tried with partial success. Better results probably would have been obtained if trap rock or other suitable aggregate had been available at Buffalo. Experiments have been made with brick linings and some other materials. It is believed that some form of concrete, or brick or ceramic material, will prove more satisfactory and less costly than steel. At the present time (1942), when steel is urgently needed for war, there is particular merit in finding some other satisfactory material. With the adjustments indicated to meet the abrasion problem, the maintenance costs on equipment will be materially reduced and better continuity of operation will be obtained.

Some of the operating results for dewatering and incineration for the three years of operation of the Buffalo treatment works are given in Table 1. Some

TABLE 1.—COMPARISON OF OPERATING RESULTS FOR DEWATERING AND INCINERATION

Item	Description	July, 1938, to June, 1939	July, 1939, to June, 1940	July, 1940, to June, 1941
1	Moisture in raw sludge (%)	96.54	93.38	92.77
2	Moisture in digested sludge (%)	87.8	89.87	90.89
3	Volatile matter in raw sludge (%)	57.6	56.4	62.9
4	Volatile matter in digested sludge (%)	41.5	42.9	47.5
	Chemicals Used per 100 Lb of Dry Solids:			
5	Ferric chloride	2.31	3.10	2.93
6	Lime (Ca O)	9.25	11.62	11.21
7	Moisture in sludge cake	61.8	62.2	63.1
8	Volatile matter in sludge cake (%)	38.8	38.1	42.9
9	Filter rate, in lb of dry solids per sq ft per hr	....	7.58	6.77
10	Incineration; total furnace hours	....	9,231	10,278
11	Dry solids burned (1,000 lb)	13,559	23,525	23,226
12	Solids burned per furnace hour	....	2,550	2,260
13	Gas used for incineration, in cu ft per lb of dry solids burned	....	1.30	1.23
14	Average daily gas production (1,000 cu ft)	....	476	516
15	Heat content of sludge cake (Btu per lb)	....	3,790	4,320

of the developments in these operating results are worthy of note. The relatively high moisture content in raw sludge during the first year was largely due to inexperienced personnel and changing of operators. As the operators became more skilful in handling sludge and operating sludge pumps, they gained a normal control of the condition of the raw sludge. The increase in the volatile matter of raw sludge and digested sludge was apparently due to a decrease in the amount of grit carried by the sewage. During the first year of operation there was an extensive program of sewer cleaning that probably created a steady flow of grit to the treatment works. There has been some decrease in the solids burned per furnace hour. The heat content of the sludge cake has showed a marked increase, and there has been a corresponding decrease in the amount of gas used for auxiliary fuel. Computations show that the total quantity of heat dissipated by the furnaces has remained practically constant even with the decrease in solids burned.

Actual expenditures for operating the sewage treatment works for the fiscal year 1940–1941, together with costs per million gallons, are given in Table 2. A summary of costs, according to the processes involved in sewage disposal at Buffalo, is given in Table 3.

TABLE 2.—OPERATING UNIT COSTS; SEWAGE DISPOSAL AT BUFFALO, N. Y., 1940–1941

Item	Description	Total cost	Cost per million gallons
	Pumping:		
1	Outlying pumping stations	$ 22,376.94	$0.44
2	Main pumping station	45,885.70	0.89
3	Administrative expense	36,121.53	0.70
4	Laboratory	25,451.82	0.49
5	General sedimentation	50,959.68	1.00
6	Chlorination	48,633.60	0.95
7	Sludge disposal and heating	95,242.90	1.86
8	Maintenance and repair	61,656.59	1.20
9	Yards and grounds	31,916.46	0.62
10	Miscellaneous	4,648.72	0.09
11	Total	$422,893.94	$8.24

TABLE 3.—SUMMARY OF OPERATING COSTS BY PROCESSES; SEWAGE DISPOSAL AT BUFFALO, N. Y., 1940–1941

Item	Description	Pumping	Primary treatment	Sludge disposal	Total
1	Direct cost[a]	$68,262.64	$50,959.68	$95,242.90	$214,465.22
2	Administration	8,320.00	12,641.53	15,160.00	36,121.53
3	Maintenance and repairs	6,166.00	21,580.59	33,910.00	61,565.59
4	Miscellaneous	7,320.00	14,625.18	14,620.00	36,565.18
5	Laboratory	.....	12,725.91	12,725.91	25,451.82
6	Chlorine	.....	.....	.....	48,633.60
7	Total	$90,068.64	$112,532.89	$171,658.81	$422,893.94

[a] Salaries and supplies.

The total flow for the year was 51,262 million gal, and the total quantity of solids removed as raw sludge (dry basis) and grit was 18,300 tons. From these data the following unit costs are obtained:

Total cost of treatment, in dollars per million gallons	8.25
Cost of treatment, not including pumping and chlorination, in dollars per million gallons	5.97
Cost of treatment, in dollars per ton of raw solids (dry basis) disposed of	23.50
Cost of treatment, not including pumping and chlorination, in dollars per ton of raw solids	16.70
Total cost of solids disposal, in dollars per ton of raw solids disposed of	9.38
Cost of solids disposal, including only direct costs and maintenance and repairs, in dollars per ton of raw solids disposed of	7.07

Summary

Operating experience in the disposal of sludge from the Buffalo primary sewage treatment works has led to developments in the selection of materials and in the use and arrangement of equipment. Some of the developments indicated may be briefly summarized as follows:

1. Sludge tanks have proved valuable to take up the irregularities in daily sludge quantities, to provide flexibility in the operation of incinerators, and to supply gas for fuel.
2. In applying chemicals for sludge conditioning, the tendency has been toward the use of manual rather than automatic devices, and arrangements which permit the operator to readily observe rates of chemical application.
3. Canton flannel has proved most satisfactory as a filter cloth.
4. A sludge-feeding device between the filter and the incinerator has been useful, but there is still some question whether it is worth its cost.
5. Muriatic acid has proved satisfactory and economical for cleaning lime scale from filter screens and pipe lines.
6. Although at Buffalo the major problem in sludge drying and burning has been the abrasion of equipment, it appears that this problem may vary widely in severity in different plants. The variation is probably due to differences in sludge characteristics and may also be affected by the nature of auxiliary fuel used.
7. Maintenance costs due to abrasion may be minimized in general by placing fans on the "clean air" side of cyclones, by using minimum speeds on moving parts, and by using concrete or ceramic linings instead of steel on stationary parts subjected to abrasion.
8. Operating results over the first three years show reduction in moisture content of raw sludge and increases in the moisture content, volatile matter and heat content of digested sludge and sludge cake. Incinerators have maintained their rate of heat dissipation.
9. Operating costs have amounted to $8.24 per million gallons treated, including pumping and chlorination, and direct costs of sludge disposal have amounted to $7.07 per ton of raw solids.

VACUUM FILTERS USED IN PRIMARY SLUDGE DEWATERING

By Ralph E. Fuhrman,[3] Assoc. M. Am. Soc. C. E.

Synopsis

The primary dewatering of digested sewage sludge by vacuum filters is one of the many mechanical processes that gained popularity in the decade 1932–1942. In the early 1930's, this service of vacuum filters was still in the experimental or pilot plant stage, whereas a very few years later saw the installation of huge filter areas. Sludge treatment literature of this period abounds with observations on vacuum filtration of digested sewage sludge and it is the purpose of this paper to indicate various experiences in the District of Columbia.

Description of Facilities

Sludge at the sewage treatment plant of the District of Columbia is dewatered by the operation of four vacuum filters, each with a filter area of 500 sq ft. These units, and the related equipment serving them, are sufficient to dewater the primary digested and elutriated sludge produced by the plant within the work week consisting of five 8-hr days. Between operating periods, sludge is held in the digestion tanks to be withdrawn as desired. After elutriation, the sludge is elevated to the filter floor by three closed impeller centrifugal pumps large enough to pass a 3-in. sphere. The sludge delivered to the mixing tank is controlled by a volumetric meter actuated by the level of sludge within the filter pans or troughs. To the required amount of sludge is added ferric chloride as a conditioning agent. Under the usual method of operation the mixing time amounts to 3 or 4 min, after which the sludge passes through rubber-lined pipes to the filter units. Provision for the addition of lime was made in the plant, but so far the use of lime has not been necessary on digested and elutriated primary sludge. As auxiliary equipment, vacuum pumps with an operating displacement of 2 cu ft per min per square foot of filter area are provided, as are filtrate pumps and low-pressure blowers, for freeing the filter cake from the cloth.

Ferric chloride is stored in two 8,000-gal, rubber-lined tanks and is pumped to elevated tanks with float switches by centrifugal pumps. The elevated tanks supply sufficient operating head for the ferric chloride feeders at the mixing tanks. Ferric chloride, which is purchased and fed as a 40% solution, is siphoned into the storage tanks from railroad tank cars. All pipe lines handling ferric chloride are hard rubber, or steel pipe lined with hard rubber, and all lines conveying conditioned sludge or filtrate are rubber-lined steel pipes.

[3] Supt., Sewage Treatment Plant, District of Columbia, Blue Plains, Washington, D. C.

Elutriation facilities provided at the plant include two air-agitated tanks for the mixing of digested sludge with river water or supernatant from the elutriation settling tanks when countercurrent elutriation is used. The unit is provided with appropriate sludge pumps and wash-water pumps necessary for the elutriation process. Wash water may be drawn directly from the Potomac River or from the plant effluent conduit. Vacuum priming facilities are provided as all of the pumps are centrifugal.

Operating Experiences

In operation of the elutriation mixing tanks, experience indicated that large quantities of fine sand passed the grit chambers and settled out of the sewage in the sedimentation tanks. This material was pumped into the digesters with raw solids and, since it was unaffected by digestion, settled out in the elutriation mixing tanks in spite of air agitation through porous plate diffusers. The mixing tanks accumulated this very fine sand until practically no mixing of the passing liquids was attained. From this observation and others the mixing of sludge and wash water seems to be of minor importance, and as a trial the inlet and outlet of one mixing tank were connected by a 12-in. pipe. In this manner, the sand that formerly settled in the mixing tank is transported gradually with the sludge to the vacuum filters. The mixing of sludge and wash water seemed to be sufficient by the flow of these liquids within the pipe line. This total time of flow is usually about 20 sec in contrast to the mixing-tank period of 15 to 20 min. Plans are being made to try a similar scheme on the other mixing tank and if successful the problem of removing this fine sand will have been eliminated.

Initially the sludge pumps provided to elevate the digested, elutriated sludge from the elutriation tanks to the mixing tanks were sufficient to pass only a 2-in. sphere and in the early service of these pumps incessant clogging was experienced to the point where continuous dewatering was impossible. After considering a number of possible solutions to the problem, including the possibility of pumps equipped with cutters, plain centrifugal pumps similar in design to those originally installed were purchased, the only differences being in the size of the impeller passage, which was 3 in. in the new pumps, and the speed, which was 1,140 rpm as a maximum.

In the first two years after installation, some of the hard-rubber pipe lines showed much sagging between the supports of the pipes. Additional supports were first provided and although this reduced the possibility of a failure in the lines they were still unsightly. However, sufficient sagging was evidenced to put extra strains on the pipes. The remedy chosen was the replacement of these lines by steel pipe with a hard-rubber lining. This pipe has completely overcome the difficulties of the original material.

An operating procedure that has been found to be very important is that of prompt and thorough washing of filters at the end of an operating period. Filters at the plant are washed for 30 to 45 min after the operating period to remove, as far as possible, particulate matter that would "set" and tend to clog or blind the filter cloth. Analyses of discarded filter cloths have revealed 52% iron salts. At the suggestion of A. L. Genter, M. Am. Soc. C. E., reclamation of cloths was attempted with oxalic acid. For this use, a 1% solution of oxalic

acid is made up in the filter pan and the filter is allowed to turn in the solution for a 24-hr exposure. For a particular cloth, for example, after 528 hr of service the oxalic acid wash was applied and service resumed until the cloth had served a total of 682 hr, an increase of about 30%. On the basis of a filter cloth cost of $120, the hourly cost of service was reduced from $0.23 to $0.18. Likewise, at a cost of oxalic acid of about $2.50 for the cloth, additional filtering service worth $35 was obtained. Some of these data have been reported previously (3).

During each summer of filter operation, fungus appeared on the filter cloths between the usual week-end shutdown from Friday afternoon to Monday morning. Within this period, the filter cloth disintegrated in spots all over the filter and rendered it practically useless. The responsible organism has not been identified, but it is probably one of the molds or bacteria known (4) to be destructive to wool.

The remedy for this attack is to apply a 100-ppm solution of copper sulfate to the filter cloths whenever the plant is shut down for more than 16 hr; that is, copper sulfate is applied every Saturday and Sunday, as well as any other nonoperating day, by rotating the filter with the solution in the filter pan long enough to saturate the wool cloth.

Before remedial measures could be developed, some cloths gave very short filtering service. The minimum life has been 32 hr of service and the maximum 899 hr. During the calendar years of 1940 and 1941, the minimum life was 227 hr, the maximum 682 hr, and the average life 408 hr. Cloths have cost from $87 to $134, with an average cost of approximately $115 each.

Value of Elutriation

P. D. McNamee has shown (5) that, without elutriation, the sludge produced at the District of Columbia plant would require about three times the

TABLE 4.—Value of Elutriation

Item	Description	Dollars
1	Cost of construction, including mechanical equipment	40,500.00
2	Patent rights	12,000.00
3	Total investment (Items 1 and 2)	52,500.00
	Fixed Charges of Elutriation Facilities:	
4	Complete unit, amortized over 20 years at 3% (0.037 × Item 1)	1,498.50
5	Interest on the investment (0.03 × Item 3)	1,575.00
6	Total, per year (Items 4 and 5)	3,073.50
	Operating Cost for the Fiscal Year Ending June 30, 1941:	
7	Labor and supervision	2,496.00
8	Materials and supplies	163.00
9	Electrical energy	252.00
10	Total, for the year (Items 7, 8, and 9)	2,911.00
11	Total annual cost of operation and fixed charges, 1941 (Items 6 and 10)	5,984.50
12	Reduction in ferric chloride required	17,000.00
13	Value of elutriation, in dollars per year (Item 12 minus Item 11)	11,015.50

ferric chloride now used. So far, it has not been practicable to operate the vacuum filters with unelutriated sludge to obtain plant data to check this value, but under the circumstances of the comparison, there is reason to believe it

would be quite close to an actual operating figure. Acceptance of this figure makes possible an evaluation of the sludge washing facilities at this plant. Table 4 indicates the cost of construction and operation of this unit. The operating costs are those for the year ended June 30, 1941, which was a full operating year. During this period, $8,500 was spent on ferric chloride for conditioning the sludge prior to dewatering. Accordingly, without elutriation, ferric chloride, if it had been the sole conditioner, would have cost three times this amount, or $25,500. The value of elutriation may be given, then, as $17,000 less the operating cost of, say, $6,000 per year, or a saving of $11,000 per year. From these data, it is apparent that the elutriation unit will pay for itself every 4 or 5 years. These conclusions are based on the condition that ferric chloride alone would be the conditioner if elutriation were not practiced. Furthermore, it is assumed that costs of filtration, as affected by filter yield and life of cloth, would be the same, though all data indicate that greater yields may be expected from elutriated than from unelutriated sludge. For this reason, additional savings made possible by elutriation arise from reduced filter area and reduced filtering time. Consideration of these advantages would increase further the monetary savings affected by elutriation.

Operating Results

During the 1941 fiscal year which ended June 30, 1941, the sludge dewatering facilities along with elutriation enjoyed a year of full operation. In the course of the year, volatile matter in digested sludge varied from 39.0% to 48.4%,

TABLE 5.—Average Results of Analyses of Elutriated Sludge

Month	Specific gravity	Solids (%)	Volatile matter in solids (%)	Wash water used, in 1,000 gal daily	Ratio of wash water to sludge volume	Alkalinity, PPM		Effective wash water ratio[a]	Power for pumping wash water (kw-hr per day)
						Digested sludge	Elutriated sludge		
1940									
July	1.04	9.2	39.0	160.9	4.5	3,499	368	8.5	35
August	1.03	8.4	37.6	204.2	4.1	3,583	450	7.0	33
September	1.03	7.9	39.1	229.9	5.0	3,486	395	7.8	37
October	1.02	6.3	45.6	160.0	4.2	3,692	390	8.5	28
November	1.02	6.2	46.6	129.6	4.3	3,428	358	8.6	22
December	1.02	6.4	45.3	81.7	3.3	3,409	399	7.5	19
1941									
January	1.03	6.1	45.9	205.8	4.0	3,708	489	6.6	33
February	1.02	5.8	45.7	226.0	2.9	3,447	564	5.1	35
March	1.02	5.6	46.8	243.9	3.2	3,232	674	3.8	35
April	1.03	6.2	45.3	221.6	3.8	2,759	382	6.2	35
May	1.02	5.2	48.4	197.1	4.5	3,044	488	5.2	29
June	1.02	5.0	47.6	192.0	3.9	3,085	440	6.0	36
Mean	1.03	6.5	44.4	....	3.8	3,364	450	7.5	....

[a] Based on alkalinity removal.

manifesting the extra load on the digestion tanks resulting from the large population growth in the area served.

Reference to Table 5 reveals the analytical and other operating data on sludge elutriation during the year. Of particular interest is the fact that 3.8 volumes of Potomac River water to one volume of digested sludge gave a total

TABLE 6.—SLUDGE DEWATERING PLANT OPERATION

Month	Days of dewatering	Filter hours	Cake produced, in tons (wet)	Solids in cake (%)	Dry solids in cake tons	Yield of cake solids[a] (dry)	Suspended solids in filtrate (ppm)
1940							
July	21	467.7	2,370.7	32.0	758.6	6.5	95
August	19	435.7	2,752.8	30.3	834.1	7.7	90
September	20	469.1	2,786.2	29.1	810.8	6.9	41
October	17	298.0	1,513.2	27.0	408.6	5.5	38
November	14	241.7	1,203.3	26.5	318.9	5.3	48
December	15	238.5	1,281.4	27.6	353.7	8.3	39
1941							
January	20	380.9	1,880.8	28.2	530.4	5.6	29
February	18	485.9	2,121.0	28.4	602.4	5.0	13
March	21	592.1	2,549.3	26.9	685.8	4.6	13
April	18	412.0	1,861.7	31.1	579.0	5.6	25
May	18	319.6	1,610.5	28.4	457.4	5.7	37
June	19	363.9	1,839.1	28.4	522.3	5.7	35
Total	220	4,705.1	23,770.0	....	6,862.0	....	....
Mean	18	392.0	1,980.0	28.9	....	5.9	....

[a] In pounds per square foot per hour.

alkalinity reduction equivalent to 7.5 volumes of wash water. The gain represented by these figures may be attributed to counter current elutriation, which was used throughout the year.

TABLE 7.—COSTS OF SLUDGE ELUTRIATION, DEWATERING, AND DISPOSAL

Item	Description	Dollars
1	Ferric chloride (Fe_2Cl_3)	8,500
2	Other operating costs of elutriation and dewatering	23,313
3	Total (Items 1 and 2)	31,813
4	Cost of sludge disposal (railroad shipment)	20,929
5	Total operating cost (Items 3 and 4)	52,742

Table 6 shows the detailed operation of the sludge filters for the year. For the entire period, 23,770 tons of sludge cake were produced, with an average solids content of 28.9%. These 6,862 tons of dry solids were dewatered by the vacuum filters at the average rate of 5.9 lb per sq ft per hr. The ferric chloride used during the period was 218.54 tons, or an average dose of 3.18%. Costs of sludge dewatering are as given in Table 7.

TABLE 8.—UNIT COSTS OF SLUDGE TREATMENT AND DISPOSAL FOR 1941

Item	Description	General operating costs	Ferric chloride	Sludge disposal	Total
1	Per ton of sludge cake	$0.98	$0.36	$0.88	$2.22
2	Per ton of dry solids in the cake	3.40	1.23	3.06	7.69
3	Per million gallons of sewage treated	0.59	0.21	0.53	1.33

Table 8 indicates the unit costs of sludge treatment and disposal for the year. It should be emphasized that these costs are for sludge elutriation,

vacuum filtration, and cake disposal only and do not include costs of sludge digestion.

SUMMARY

At the District of Columbia Sewage Treatment Plant, elutriation of primary digested sludge prior to vacuum filtration has been found beneficial to the dewatering process and economically justified. Some simplifications in the sludge mixing facilities have been made, and measures for prolonging the life of the woolen filter cloths have been practiced.

POSSIBLE ECONOMIES IN SLUDGE DISPOSAL PRACTICE

By George J. Schroepfer,[4] Assoc. M. Am. Soc. C. E.

Synopsis

Because of the fact that considerable information has been published (6)(7) concerning the processes used and the results of operation of the sludge disposal system at the Minneapolis–St. Paul (Minn.) plant, such data as are presented herein are limited in scope and are condensed and arranged to illustrate certain points which the writer wishes to make.

The sludge disposal processes at the Minneapolis–St. Paul plant were placed in operation in July, 1938. They consist essentially of concentration, dewatering on vacuum filters, and incineration. The vacuum filters, of which there are six, each have an area of 500 sq ft. The three incinerators are of the multiple-hearth type; each has eight hearths and is 22.25 ft in outside diameter.

Operation and Cost Data at the Minneapolis–St. Paul Plant

In Table 9 are shown comparative data on sludge filtration and incineration by years. It is interesting to note that it has been possible to maintain an

TABLE 9.—Comparison of Sludge Filtration and Incineration Data by Years; Minneapolis–Saint Paul Sanitary District

Item	Description	1938[a]	1939	1940	1941[b]
1	Sewage treated (mgd)	88.9	102.9	104.2	111.9
2	Raw sewage strength, suspended sewage solids (ppm)	240	285	300	315
3	pH-value of concentrated sludge	6.0	5.8	5.8	5.7
4	pH-value of conditioned sludge	11.7	11.6	10.5	10.0
	Total Solids (%):				
5	Raw sludge	7.15	7.79	8.07	8.11
6	Thickened sludge	9.02	9.13	9.34	9.35
7	Filter cake	35.0	34.7	35.2	35.5
8	Volatile solids, filter cake (%)	52.1	59.7	59.8	58.8
9	Suspended solids, filtrate (ppm)	315	260	165	135
10	Filter cake production (wet tons daily)	172.0	303.3	308.3	305.7
11	Filter cake production (dry tons daily)	58.6	103.3	108.4	108.6
12	Filter rate (lb per sq ft per hr, dry)	5.50	4.73	4.29	3.90
13	Detention period in conditioning tanks (min)	13	12	10	9
	Conditioning Chemicals; Per Cent of Dry Sewage Solids:				
14	Lime as Ca O	10.3	5.68	4.76	3.81
15	Ferric chloride	3.17	2.10	1.92	1.60
16	Incineration, power (kw-hr per ton of dry solids)	18.5	17	17	14
17	Incineration, fuel oil (gal per ton of dry solids)	3.7	2.3	2.5	1.5

[a] July to December, inclusive, 1938. [b] January to October, inclusive, 1941.

average solids concentration in the sludge prior to filtration of more than 9% (91% moisture). The moisture content of the sludge cake has consistently averaged approximately 65%. In connection with statements that will be

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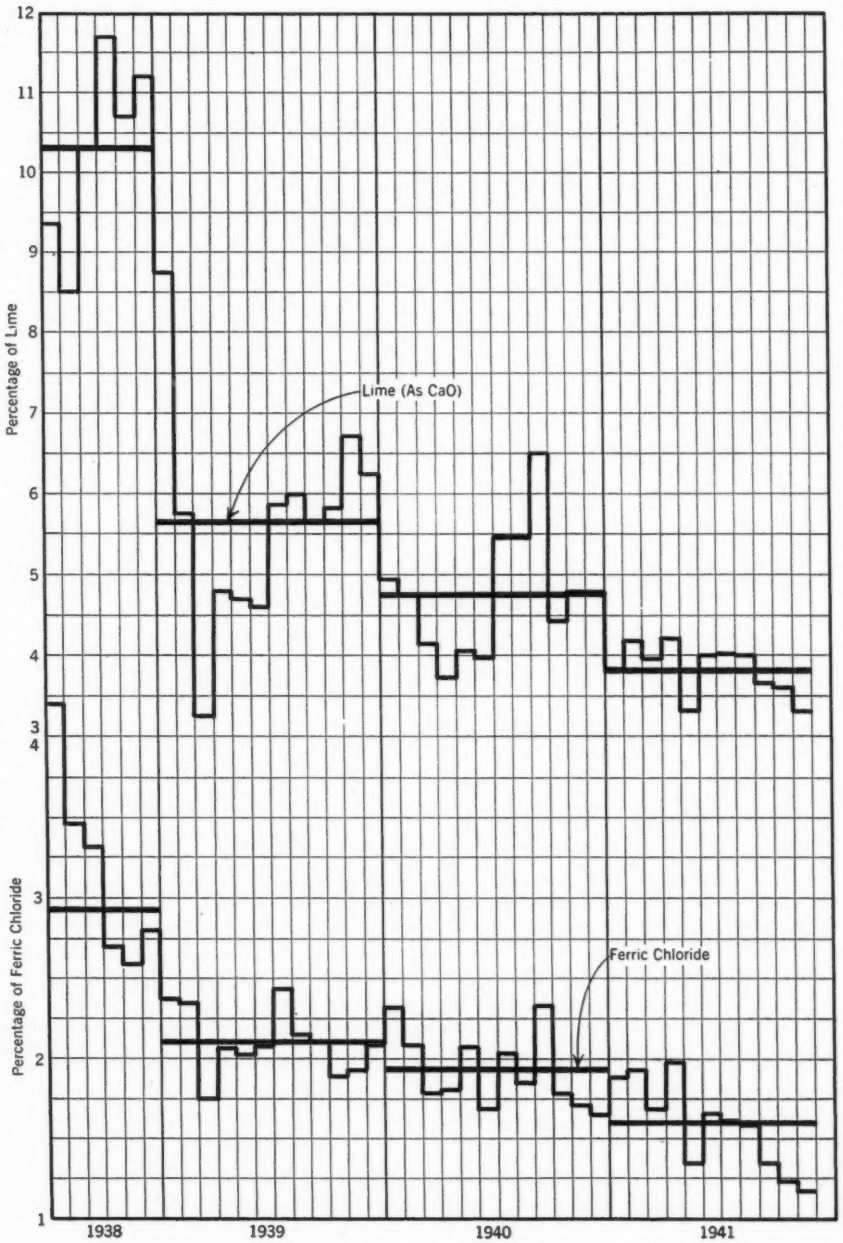

FIG. 2.—CHEMICALS FOR SLUDGE CONDITIONING, MINNEAPOLIS-SAINT PAUL (MINN.) SANITARY DISTRICT (EXPRESSED AS PERCENTAGES OF DRY SEWAGE SOLIDS)

made subsequently concerning this matter, the progressive and planned reduction in the filter rate from 5.50 lb per sq ft per hr in 1938 to 3.90 lb per sq ft per hr in 1941 should be noted. This ties in with the progressive reduction in chemicals shown in Table 9 and illustrated more graphically in Fig. 2. The reduction of chemicals to less than one half of their previous values is of some economic magnitude.

Costs of filtration and incineration for the various years of operation are shown in Table 10. The costs of operation and maintenance of the sludge

TABLE 10.—COMPARISON OF FILTRATION AND INCINERATION COSTS BY YEARS; MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

Item	Description	1938[a]	1939	1940	1941[b]
	Filtration Cost, in Dollars:				
1	Operation	47,428.74	92,177.80	76,568.91	60,744.22
2	Maintenance	2,990.74	6,932.35	5,306.02	5,706.86
3	Total	50,419.48	99,110.15	81,874.93	66,451.08
	Incineration Cost, in Dollars:				
4	Operation	19,233.60	32,393.09	35,697.84	27,007.48
5	Maintenance	1,808.60	4,691.93	14,524.63	13,522.59
6	Total	21,042.20	37,085.02	50,222.47	40,530.07
7	Dry sewage solids disposed of, in tons	10,248.0[c]	37,703.0[c]	37,319.0	31,347.0
8	Filtration cost, in dollars per dry ton	4.92	2.64	2.19	2.12
9	Incineration cost, in dollars per dry ton	2.05	0.99	1.34	1.29
10	Total cost of sludge disposal, in dollars per dry ton	6.97	3.63	3.53	3.41
11	Cost of entire project (collection, treatment, and sludge disposal), in dollars per dry ton	15.40	7.98	8.18	8.25
	Conditioning Chemicals, % of Dry Sewage Solids:				
12	Lime as Ca O	10.30	5.68	4.76	3.81
13	Ferric chloride	3.17	2.10	1.92	1.60
	Cost of Chemicals, in Dollars per Dry Ton:				
14	Lime at $9.30 per ton (Ca O)	0.96	0.53	0.44	0.35
15	Ferric chloride at $39	1.24	0.82	0.75	0.62
16	Total	2.20	1.35	1.19	0.97
17	Cost of chemicals to filter 110 tons of dry solids, in dollars per day	242.00	149.00	131.00	106.70

[a] July to December, inclusive, 1938. [b] January to October, inclusive, 1941.
[c] Includes conditioning chemicals.

disposal processes have been decreased from $6.97 per dry ton in 1938 to $3.41 in 1941. It is of interest to point out the relative magnitude of the cost of sludge disposal (which in this project has been between 40% and 45% of the total cost), as compared to the total cost of the collection, treatment, and sludge disposal processes. As shown in Table 10, the cost of chemicals for sludge conditioning has decreased progressively from $2.20 to $0.97 per dry ton. These are based on present chemical costs, which are somewhat lower than early costs. The magnitude of these reductions is shown in the daily costs to filter the present quantities of sludge. From a value of $242 per day in 1938, the costs have been reduced to $106.20 in 1941. This amounts to a total annual reduction of $50,000.

Improvements in Filtration and Incineration as Applied to the Minneapolis–St. Paul Project

In order that the remarks made herein can be evaluated properly in the light of definite conditions, certain improvements and operating goals as they apply to this project are mentioned briefly. This has the further advantage of restricting the observations to certain particular conditions, since the inconsistency of sludge behavior characteristics makes general observations inapplicable or even misleading. For this reason improvement changes effected at this plant should not be considered as "cure-alls," but rather as possibilities for exploration and development as they might apply to a different plant.

Concentration of Sludge.—It is a commonly recognized fact that, within limits, the more concentrated the sludge that can be produced the greater will be the efficiency of vacuum filtration, since, with a concentrated sludge, the gallonage of liquid sludge per ton of dry solids is reduced, resulting in decreased chemical costs and less work for the vacuum filters. The concentration of sludge in the Minneapolis–St. Paul plant is controlled by weighing a known quantity of sludge that is pumped from the sedimentation tanks at frequent intervals, as well as by soundings and determinations of moisture content of sludge in the tanks. The moisture content is further reduced by the control of the concentration tanks in which a further separation and concentration are effected by reason of the withdrawals of supernatant liquor. That such procedures are effective is shown by the fact that the moisture content of the sludge as removed from the settling tanks averaged 8.1% in 1940 and 1941, with an increase in solids, by concentration, to an average of 9.35% (see Table 9).

Conditioning of Sludge.—Considerable improvement in the conditioning of sludge, and resulting reduction in the cost of filtration, can be effected by improving the process of mixing in the conditioning tanks. In the case of air mixing, which is used at the Minneapolis–St. Paul plant, better results were obtained by increasing the number of points through which air is admitted to the sludge for mixing and by decreasing the pressure and violence of the mixing. In addition, the path of the sludge, which formerly passed through the tanks with over and under baffles, was modified to create a sinuous path, both vertically and horizontally, which procedure resulted in improved mixing. Furthermore, the detention period in the conditioning tanks has been reduced from 13 min in 1938 to 7 or 8 min at the close of 1941.

Instead of adding the chemicals to the tanks through one or two pipes, riffle boards for both lime and ferric chloride have been installed. This has resulted in considerably improved dispersion of chemicals.

Suitable chemical feed equipment must be provided with an adequate range and careful checking to keep the machines in accurate adjustment. In this connection, it is desirable to make tests with different chemicals to protect the plant against the inability to secure the chemicals regularly used. Instead of lime and ferric chloride, which are the usual conditioning chemicals, plant scale tests have been conducted using ferric sulfate and lime, and ferric chloride alone. As a safeguard, a quantity of ferric sulfate has been stored for more than a year. Although not as economical as the present combination (approximately 25%

more expensive), the usefulness of substitute chemicals in emergencies has been demonstrated.

Filter Mediums.—The type of cloth used as a filter medium should be considered. After some, but not extensive, experimentation, 8-oz Canton flannel has been adopted at this plant. However, the difficulty of obtaining this material during wartime, and its increased cost, have necessitated additional investigations leading to the possible use of a new cloth type. The life of filter cloths is also an important subject for investigation. For experimental purposes, cloths have been used for as long as 800 hr, but such procedures are distinctly not economical for the type of sludge disposed of at the Minneapolis–St. Paul plant. Newer cloths give somewhat higher yields for the same quantity of chemicals and, conversely, require less chemicals for the same yield. With high chemical dosages, the importance of this factor is not readily apparent, but with dosages approaching 1% of ferric chloride and 3% of lime, such factors show up quite markedly. In spite of the fact that the cloth is structurally in good condition, its use has been restricted to 350 hr. In recent months, because of increased cost of cloths, a longer period of usage has been adopted. When it is realized that cloth and wire constitute only approximately 3% to 10% of the cost of chemicals, the economy of maintaining satisfactory cloth will be readily apparent.

Tests have been made on a plant scale to demonstrate the effect of cloth age on filter yields. As a result of such tests, it has been shown that there is a loss in yield of 2.5% for each 100 hr of additional cloth life. Thus, at 500 hr, the yield of a filter is approximately 10% to 15% less than that of a new cloth, other conditions being constant. These results apply to an interval of 4 hr between washings (which is current practice at this plant). With longer intervals between washings, the yield with low chemical dosages falls off rather rapidly for older cloths. In fact, even after the first hour or two, the yield of older cloths drops off.

The aforementioned tests were conducted in an investigation to determine economical cloth life. The results of this study for present Minneapolis–St. Paul conditions are as summarized in Table 11. This comparison is based

TABLE 11.—COMPARISON OF DAILY COSTS OF FILTER CLOTH AND CHEMICALS (DOLLARS)

Description	CLOTH LIFE, IN HOURS							
	100	200	300	400	500	600	700	800
Wire and cloth	36.00	18.00	12.00	9.00	7.20	6.00	5.15	4.50
Chemicals	73.00	75.50	78.00	80.00	82.00	84.00	86.50	89.00
Total	109.00	93.50	90.00	89.00	89.20	90.00	91.65	93.50

on present cloth costs, in 50-yd bolts, of $0.235 per sq yd, and indicates the economical life to be about 400 hr. At different cloth prices, such as the previous price of $0.135 per yd, or with varying chemical unit prices or required additions, the economical balance, of course, would vary.

Regardless of the age of the cloth, an appreciable improvement can be effected by proper washing. It has been found that the washing can be improved considerably by rearranging the washer sprays to impinge along the filter drums almost at right angles to the drums, instead of tangentially, as in the usual installations. However, regardless of the efficiency of washing, certain material remains in the cloth even after extended periods of washing. Analysis of the blinding material in a number of cloths after approximately 300 hr to 400 hr of service indicates that approximately 60% is calcium carbonate and approximately 20% is grease. At the present price of cloth, it appears to be economical to give each cloth an acid bath, using one to two carboys of hydrochloric acid (in 600 gal of water), together with an inhibitor, after approximately 350 hr of service, thereby reducing the blinding and increasing the economic cloth life to approximately 500 hr to 600 hr. This expedient has the further advantage of providing an inexpensive means of removing calcium carbonate from the screens and splines of the filters, which formerly became so incrusted with material as to interfere with the free passage of water through the filter. This material was previously removed by sand blasting and other mechanical means, but because of savings in time and expense, and for other reasons, the chemical method of removal is more desirable.

It has been found economical, in the case of the Minneapolis–St. Paul plant, to wash filters frequently. During the first several months of operation, washing was confined to intervals of approximately 24 hr or longer. This was changed later to at least one washing of each filter every 8 hr, and in the last several months a washing has been required at least every 4 hr. This has marked effects on the quantity of chemicals required. In fact, the importance of maintaining a clean filter medium has resulted in the development, by the staff of the Sanitary District, of a method of washing filters while they are in service, either continuously, or at frequent intervals, as dictated by the relative economics. Although this method has not been incorporated in this plant to date (1942), because of its recent conception and the effect of priorities on procurement of the necessary material, its simplicity, and therefore its likelihood of operating satisfactorily, justify a description herein. Instead of its usual location, on the far side of the filter from the take-off plate, the wash-water pipe and nozzles would be installed immediately below the top edge of the take-off plate, so that the washing operation would be done in the section of the drum between the take-off and the submergence in the sludge, which section is not utilized in the usual installation. A trough suspended immediately above the sludge level would return the spent wash water to the overflow box on the filter. It is expected that, in addition to the advantages gained through maintaining a clean filter medium, the installation of this method of washing, in effect, will result in the addition of one filter, since it is necessary to have one filter in reserve a major part of the time to take the place of filters being washed.

Economic Speeds of Rotation of Filter Drums.—It is commonly recognized that, within limits, a change in speed of the filter drum increases or decreases the filter yield. It is possible to determine for each size of installation and type of sludge the economic range of speeds at which the filters ordinarily should be operated.

Adequate Filter Area.—The filter rate has a marked effect on the economics of filtration. Where incineration or drying is the practice, it is desirable to have a multiplicity of units so that uniform feed to subsequent units can be maintained at all times. An important loss of filter time results from washing of the filters and, to hold feeds constant, it is advisable to have a spare unit, in addition to the regular operating filters, to use during washing of regular units.

Return of Incinerator Ash.—Other papers by the author mention the advantage of returning incinerator ash to the raw sewage after filtration and incineration. This possibility, as well as a variation of it—adding ash to the sludge immediately prior to conditioning—bears investigation.

Incineration.—The desirability of maintaining a uniform feed of solids to the incinerator has been mentioned. By doing so, it is possible to improve operation and reduce maintenance cost. Sudden and large changes in sludge feed result in rapid changes in furnace temperatures, with consequent long-time damage to refractory materials.

As an experiment, the preheater has been removed on one of the three incinerators, which results in considerable savings in power (which constitutes 25% of incinerator costs) and maintenance costs. Although the original preheater was still serviceable, the result of investigations seemed to justify removal of the preheater from one furnace. This was done in May, 1941. Operation of this unit indicates a marked reduction from 17 kw-hr to 5.5 kw-hr in the quantity of power required. Operation of the unit over a period of six months has justified similar changes in the other two units at an estimated savings in power, replacement, and maintenance costs of $12,000 annually. This will result in a reduction in operation and maintenance costs of incineration of approximately 33¢ per ton.

Attention is called to the reduction in the quantity of fuel oil required by the incinerators (Table 9). Only 1.5 gal of fuel oil are now required per ton of dry solids incinerated, this quantity being required in heating up, cooling down, and holding temperatures in incinerators not in operation. A substantial savings in this item was effected by the installation of duct work so that the heat available in operating furnaces could be utilized to heat up, cool down, or maintain temperatures in any desired unit. In view of the fact that fuel costs are a minor item in sludge disposal expense at this plant (averaging 5¢ to 10¢ per dry ton), these savings are not of major importance, but are cited to indicate the possibilities available.

An interesting possibility relates to the utilization of the heat resulting from incineration. Various methods have been developed which indicate the possibility of generating sufficient steam for all heating purposes and electric power requirements.

In general, it can be stated that the possibility of effecting major economies and that the operating difficulties encountered are somewhat less in connection with sludge incineration as compared to vacuum filtration. The difficulty of making marked savings in incineration costs is evident from the fact that approximately 50% of the cost is labor, of which the major part is the time of one man on each shift to operate the incinerators.

Effect of Filter Rate on Cost of Filtration

In the past, it was erroneously believed that the measure of the efficiency of sludge filtration was the thickness of the filter cake produced, a heavy mat being considered highly desirable. This mistaken impression has sometimes resulted in the use of higher filter rates in both design and operation than are economical. As an example, a number of plants are operating with filter rates of 5 to 10 lb per sq ft per hr. In some cases the installed area and filter size preclude the use of lower rates by the operator. Because of its importance to filter costs, the writer devotes considerable space to a discussion of this factor.

In the early period of operation of the Minneapolis–St. Paul plant, when chemical additions of about 3% of ferric chloride and 10% of lime were utilized, filter rates of 8 to 10 lb per sq ft per hr were common, the usual cake thicknesses being between 0.5 in. and 0.75 in. In addition to causing other difficulties (which are enumerated in earlier papers by the author), this type of operation was distinctly not economical. As shown in Table 12, filter rates were pro-

TABLE 12.—Effect of Filter Rate on Requirements of Chemicals for Sludge Conditioning

No.	Description	(*a*) No. of Filters Used (Test on Plant Scale)[a]			(*b*) Average Results, by the Periods:			
		6	4	3	1939	1940	1941[b]	Oct., 1941
1	Filter rate (lb per sq ft per hr)	3.00	4.60	6.15	4.73	4.29	3.90	3.55
2	Cake thickness (in $\frac{1}{32}$ in.)	5	8	12	8	7+	7−	6
	Chemical Requirement:[c]							
3	Ferric chloride	0.89	2.33	2.35	2.10	1.92	1.60	1.23
4	Lime as Ca O	2.63	5.03	7.55	5.68	4.76	3.81	3.60

[a] For a production of 110 tons of dry solids per day. [b] January to October, 1941, inclusive.
[c] Percentages by Dry Weight of Sewage Solids.

gressively reduced so that by October, 1941, the rates were only 3.55 lb per sq ft per hr. The attendant reduction in chemical requirements is also shown, but it should be emphasized that not all of this reduction can be attributed to the reduction in rate, as certain improvements and better operation were accomplished simultaneously. The test referred to in Table 12 was made during one day of operation with other influencing factors maintained constant, so that the effect of filter rates on chemical costs could be observed. The influence of filter rates on costs of operation and maintenance are shown in Table 13 and on total annual charges in Table 14. Referring to Fig. 3, it will be observed that chemical costs at high filter rates far outweigh other affected cost items, whereas at low rates the reverse is true. Conditions at the Minneapolis–St. Paul plant indicate that the economical filter rate is approximately 2 lb per sq ft per hr, based on total annual charges (including fixed charges). Considering operation and maintenance costs only, however, the economical rate is approximately 1 lb per sq ft per hr. It will be observed that the operation and maintenance costs at a 2-lb rate are less than half of what they would be at a 7-lb rate. Based on the tonnages produced at the Minneapolis–St. Paul plant, a

reduction in the filter rate from 7 lb to 2 lb per sq ft per hr would result in a saving in operation and maintenance costs of approximately $125 per day, or about $45,000 annually. Expressed in another way, the installation of one more filter in this plant would result in savings in operation and maintenance

TABLE 13.—OPERATION AND MAINTENANCE COSTS, MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

(Based on the Production of 110 Tons of Dry Sewage Solids per Day; Operation Labor Not Included)

Filter rate (lb per sq ft per hr)	Filters required	Chemicals				Costs, Except Chemicals[b]					Total Costs	
		Quantities[a]		Costs								
		Lime as Ca O	Ferric chloride	Dollars per dry ton	Dollars per day	Power	Maintenance	Changing cloth and wire	Washing	Total, Cols. 6 to 9	Cols. 5 and 10	Dollars per dry ton
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
2	9	2.30	0.85	0.55	60	25.90	7.40	15.60	1.35	50.25	110.25	1.00
3	6	3.40	1.25	0.80	88	25.10	4.95	10.50	0.90	41.45	129.45	1.18
4	4.5	4.50	1.65	1.07	118	24.80	3.70	7.85	0.70	37.05	155.05	1.41
5	3.6	5.70	2.05	1.34	147	24.40	2.95	6.10	0.55	34.00	180.00	1.64
6	3	6.80	2.50	1.62	178	24.20	2.45	5.20	0.45	32.30	210.30	1.91
7	2.5	8.00	2.90	1.87	206	24.00	2.10	4.35	0.40	30.85	236.85	2.15

[a] Percentage of dry weight. [b] Dollars per day.

of $14 per day, or approximately $5,000 per yr, which would pay for the additional investment in two to three years. The magnitude of the costs involved compels attention to this important phase of filtration economics.

TABLE 14.—TOTAL ANNUAL CHARGES, IN DOLLARS; MINNEAPOLIS–SAINT PAUL SANITARY DISTRICT

(Based on a Production of 110 Tons of Dry Sewage Solids Daily)

Filter rate (lb per sq ft per hr)	Area of filters required (sq ft)	First cost of equipment installed	Total cost of filters and structures	Annual fixed charges (6%)	Dollars per Day				Total charges[c]
					Fixed charges	Chemicals[a]	All other[b]	Total	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	9,000	250,000	370,000	22,200	61.00	30	75.00	166.00	1.51
2	4,500	130,000	240,000	14,400	39.50	60	50.25	149.75	1.36
4	2,250	75,000	145,000	8,700	23.80	118	37.05	178.85	1.63
6	1,500	55,000	105,000	6,300	17.30	178	32.30	227.60	2.07

[a] From Table 13, Col. 5. [b] Other differential operation and maintenance costs from Table 13, Col. 10. [c] Dollars per dry ton.

What has been stated concerning this matter is subject to the following local and practical limitations: It is based on the conditions (type of sludge, chemical costs, etc.) existent at this plant. A comparable analysis can, and should, be made by other operating plants, based on the conditions obtaining there, to determine (*a*) the economics of operation of the present installation, (*b*) the

possibilities of affecting economics by the enlargement of present facilities at some future date, and (*c*) economical rates for different types of sludges for the benefit of designers of future plants. Other practical limitations relate to the possible inability of present equipment to remove cake when it becomes exceedingly thin, although that limitation would not be effective at the economical value of 2 lb per sq ft per hr ($\frac{1}{8}$-in. thickness) determined herein. Furthermore, the cost of operation labor (maintenance labor and labor for changing cloth have been assumed to vary) has been assumed constant in this analysis, since it is quite definite for this plant that the same personnel which now operates six filters could operate eight filters (approximately the 2-lb rate), which number is dictated by the present size of this plant. Of course, the same number of men could not operate eighteen filters (1-lb rate) as would be required to operate three filters (6-lb rate). However, the usual decision in an operating installation will be on the question of whether one filter or, at most, two filters should be added. Therefore, usually no increase in personnel will be involved, since a large part of the operation labor is affected by the tonnage of material handled, which is assumed constant.

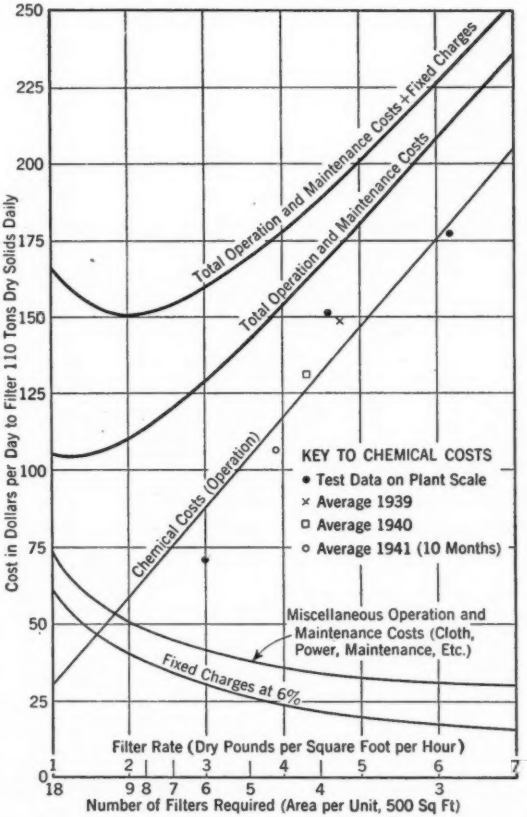
Cost in Dollars per Day to Filter 110 Tons Dry Solids Daily
250
225
200
175
150
125
100
75
50
25
0
Total Operation and Maintenance Costs + Fixed Charges
Total Operation and Maintenance Costs
Chemical Costs (Operation)
KEY TO CHEMICAL COSTS
• Test Data on Plant Scale
× Average 1939
□ Average 1940
○ Average 1941 (10 Months)
Miscellaneous Operation and Maintenance Costs (Cloth, Power, Maintenance, Etc.)
Fixed Charges at 6%
Filter Rate (Dry Pounds per Square Foot per Hour)
1 2 3 4 5 6 7
18 9 8 7 6 5 4 3
Number of Filters Required (Area per Unit, 500 Sq Ft)

FIG. 3.—DAILY COST OF MATERIALS AND SUPPLIES, POWER, MAINTENANCE, ETC., TO PRODUCE 110 TONS OF DRY SOLIDS PER DAY

COSTS OF DEWATERING AND INCINERATION AS COMPARED TO OTHER PROCESSES OF SLUDGE DISPOSAL

The relative complexity of efficient dewatering by vacuum filters presents again the question of the relative economy of this process of sludge disposal as compared to others available, particularly for the comparatively small plants. Sludge digestion and drying on sand beds are still satisfactory processes and should receive consideration in a decision on the most advantageous type

of sludge disposal for a particular project. A number of factors, including practical and economic ones, usually will influence the decision in favor of dewatering, followed by drying or incineration, in large projects. In the case of very small plants, the decision is likewise comparatively simple in favor of the older type of processes. Dependent on the type of treatment by which the sludge is produced, and other factors, the decision in the case of plants serving populations in the range from 50,000 to 200,000 should be based on a thorough analysis of the many influencing and controlling factors involved. The quantities of material to be disposed of, as they influence the likelihood of securing efficient and economical operation of a more complicated mechanical and chemical process, must be considered. To expect that even the most conscientious operator of a plant in a community of, say, 25,000 people, in which 1 to 2 tons of dry solids must be disposed of daily, can even approach the efficient and controlled operation of a plant handling 50 or more tons daily is erroneous. Extending this idea further, a plant of the size of the Minneapolis–St. Paul plant handles quantities in a single week that the small plant referred to would encounter in a year or two of operation.

For illustrative purposes the costs of various sludge disposal processes are mentioned briefly. A. J. Fischer, Assoc. M. Am. Soc. C. E. (8), has estimated the costs of different sludge disposal processes as shown in Table 15. These

TABLE 15.—COSTS OF SLUDGE DISPOSAL, IN DOLLARS PER DRY TON

Population served	DIGESTION AND DRYING ON BEDS		DIGESTION, FILTRATION, AND INCINERATION	
	Operation and maintenance	Total annual charges[a]	Operation and maintenance	Total annual charges[a]
100,000	3.40	7.00	7.10	11.50
200,000	3.50	6.70	6.80	9.80

[a] Includes fixed charges on the investment at 7.5%.

are cited to show that, at the very least, an economic question is involved in medium-sized plants. Using actual costs as reported by the late H. F. Ferguson and W. H. Wisely, Assoc. Members, Am. Soc. C. E. (9), for communities using digestion and drying in a population range from 47,000 to 200,000 (to which range the writer makes special reference), the costs of operation and maintenance vary from $2.45 to $5.10 per dry ton (higher values for the smaller plants) and the total annual charges vary from $7.85 to $12.50 per dry ton. When it is realized that the operation and maintenance costs alone for several considerably larger plants using dewatering on vacuum filters, followed by incineration, range from $5 to $15 per dry ton, the point to which the writer alludes will be evident.

Personnel Required.—The greater complexity of sludge filtration and incineration processes, both from mechanical and chemical standpoints, as compared with the older processes available, requires a higher quality of personnel for economical operation and maintenance. Of course, it is possible to filter sludge with large dosages of conditioning chemicals, and to incinerate sludge with high

maintenance charges, but to perform this work economically requires a relatively high degree of skill. New problems are constantly arising and new possibilties must forever be explored. Whatever success the Minneapolis–St. Paul Sanitary District has had in economical operation of its sludge disposal processes is attributed largely to the advantage of having a relatively large number of professional engineers and chemists in supervisory positions.

It is not enough, however, to have men with technical training in charge. It is necessary that the interest and conscientiousness of the entire personnel, including especially the filter and incinerator operators, be maintained at a high level at all times. For example, as a means of awakening the interest of the filter operator, such possibilities as control of filtration by Buechner tests, frequent calculation of chemical dosages (as often as hourly), and shift competition on chemical requirements should be utilized. Operation should never be considered to have reached what appears to be the ultimate, so that standardization in practice, with consequent stagnation, will take place. New ideas (and there can be many) must be explored and developed continually, so that the entire staff, from the superintendent down, never loses sight of the possibilities of improvement. After all, a routine standardized task is certain to become uninteresting.

Dependability of Operation.—To accomplish the purposes for which a sewage treatment plant is constructed, it is necessary that the sludge disposal processes be able to handle the raw material produced as necessary. Digestion or a sufficient period of storage prior to filtration provides some leeway for the unexpected failures of equipment. Duplicity and multiplicity of units can be secured in the larger plants, but in the smaller plants this is sometimes not considered to be economically feasible. In addition to the economics of selection, dependability is another factor that must be taken into consideration in the decision between digestion and drying on sand beds and the mechanical processes. With from three to as many as several dozen vacuum filters in a larger plant, dependability as a factor is reduced in importance by thoughtful design; but in the smaller plants that are provided with only one vacuum filter and one drying or incineration unit the operator must be eternally vigilant to take advantage of all favorable opportunities to place his equipment in such condition that it will survive the period when its operation will be most in demand. Essentially, the point that the writer wishes to make is that, although operators of plants using digestion and drying may complain of the perversity of the bacteria in the digestion tanks, or the unpredictableness of the weather as it affects drying on beds, their problems dwindle into insignificance in comparison to those of the superintendent of a small plant with complicated and numerous mechanical devices which, even with proper care, display a persistent propensity to break down at the most inopportune times.

The statements concerning the efficiency and dependability of various processes of sludge disposal are made, keeping in mind certain limitations in their applicability. In large plants the decision is quite likely to be in favor of the mechanical processes, and properly so. In the small and medium-sized plants, thorough consideration must be given to all factors, including those peculiarly local factors which influence the decision. In the very small plants

the writer feels that consideration should be given to the position of the operator, who is in the embarrassing position of complying with the requirements of public health officials as to dependable and satisfactory operation on the one hand, and meeting the demands of local public officials as to the economical operation on the other. The writer realizes, also, that all equipment improves in dependability and economy with development, and this has been the case with sludge disposal equipment. Furthermore, it is more than likely that, with careful observance of actual operation of equipment, along with more attention to details of design as they pertain to the smaller plants, on the part of designing engineers and equipment manufacturers, and that, with the present tendency to raise the standards of qualifications of small plant operators, mechanical processes, in the future, will be shown to be economical and dependable, even for the smaller plants.

Relative Costs of Filtration and Incineration

A review of the costs of filtration as compared with incineration at several plants will indicate the relatively large proportion of costs devoted to dewatering prior to incineration. The writer feels that with present practice too much of the cost of sludge disposal is devoted to the preliminary processes which serve simply to prepare the material for incineration. In spite of appreciable economies effected in filtration costs, the costs of dewatering at the Minneapolis–St. Paul plant are still approximately twice the costs of incineration. At several other plants this relation is still greater. As previously indicated, the possibility of savings in filtration is greater than in incineration. Developments in sludge disposal practice are likely to be along the line of reducing or eliminating the preliminary expenses. As a matter of fact, one or two small plants have been incinerating liquid sludges for some time. In connection with the studies of type of sludge disposal for the Minneapolis–St. Paul plant, the writer concluded in 1935 that, if the moisture content of the sludge could be reduced to 85% or 90%, dewatering on vacuum filters could be economically dispensed with, assuming that equipment was available for incinerating this type of sludge. The Sanitary District has begun experiments in which a portion of liquid sludge will be added to the filter cake in the incinerators as a means of determining the possibilities of this procedure.

Filter Rate Index

Because of the importance of the filter rate on the cost of filtration, it is desirable to use a more accurate index of filter rates than the commonly used unit of pounds per square foot of installed filter capacity per hour. This does not furnish an accurate measure of the number of square feet of filtering surface utilized in a given period, since it does not take into account varying filter speeds (changed quite frequently to hold uniform tonnage) which, of course, affect the filter area. Filter speeds used in two different plants may easily vary by as much as 50%. For this reason the writer suggests the use of the index, "pounds per square foot," based on actual filter surface used. To make the use of this unit possible, it is necessary only to install inexpensive revolution counters on the filter drums. Using the revolutions in a given period of time,

the tonnage produced in that same period, and the area of the filter, it is possible to obtain an accurate filter rate for comparisons within a plant and with other plants. Incidentally, this furnishes a readily applied method of determining when it is desirable to change filter cloths, in terms of actual work done by the filter.

Summary and Conclusions

The writer has made certain observations and indicated certain possibilities of improving operations or effecting economies in sludge disposal processes. The presentation of this paper has afforded an opportunity to state certain general impressions that the writer has had for some time. He hopes that the discussions and considerations by others in this field will serve to clarify the relation between, and the applicability of, the various sludge disposal processes available, and thereby take advantage of the temporary lull in sewage disposal construction which will precede the period of expansion in the postwar era. The impressions of the writer are summarized as follows:

(1) Consideration should be given, in existing and projected plants, to the possibility of effecting economies by the utilization of lower filter rates than have heretofore been the general practice.

(2) A closer study and a better explanation for some of the occurrences in vacuum filtration can be made by the use of a filter-rate index, based on pounds per square foot of filter surface utilized. The index commonly used does not take into account the variation in filter speeds and other factors of importance in a close scrutiny of filtration characteristics.

(3) Since the decision between the mechanical processes and the older methods of sludge disposal is usually quite clear cut in the very large, as well as in the very small, plants, the writer suggests increased consideration in selection, in the case of medium-sized plants. Certain reasons are offered for the present inapplicability of the mechanical processes to the smaller plants.

(4) The writer feels the necessity of a higher degree of directive ability, for economical operation and maintenance of mechanical sludge disposal processes, than has heretofore been considered necessary. Even in the very small plants this should approach that of professional grade. In addition, the interest of all members of the staff must be promoted continually to improve the position of mechanical processes of sludge disposal.

USE OF SEWAGE SLUDGE AS FERTILIZER

By LeRoy W. Van Kleeck,[5] Assoc. M. Am. Soc. C. E.

Synopsis

The disposal of the settleable solids or sludge produced by sewage treatment plants is a major operating problem. Air-drying, incineration, use as fill, and other means of disposal for sludges have been discussed by others. The writer's plea is for more general use of sewage sludge, properly prepared as a soil conditioner, as well as for its fertilizer value. Failure to make use of it is a waste of a natural resource.

Introduction

One bomb dropped on London in 1939 was not released by a German war plane but by Sir Albert Howard, an eminent English agricultural scientist, who, in a paper (10) presented before the British Institute of Sanitary Engineers stated:

> "When the problems of waste disposal as a whole are considered, it is difficult to escape the conclusion that the West has completely lost direction. Why are the methods of the forest, in dealing with its wastes, more effective than those of the various Councils in our large cities? The answer is obvious if we study Nature's round—the wheel of life * * *. Trouble inevitably follows any attempt to short-circuit or to interrupt this wheel. This is always followed by disease in crops, in live stock and in mankind * * *.
>
> "The farmers of forty centuries, the cultivators of the East, the Chinese in particular, have closely followed the methods of the forest when dealing with their wastes. Nothing is ever thrown away. Vegetable and animal residues are mixed and converted into humus before being added to the land, so that the soil is not overworked by having to decay this material and grow a crop at the same time * * *. The consequence is that countries like China and Japan maintain over 2,000 people to the square mile as compared with about 60 in the United States. The Far East has been doing this for centuries without any loss of fertility. After less than one hundred years, the soils of the United States are almost worn out and a great national effort is now in progress to restore the lost fertility. You have all heard about the dustbowls and the floods of the Mississippi Valley—both the inevitable consequences of improper methods of agriculture which have taken no account of the correct utilization of wastes.
>
> "In Great Britain we are rapidly following in the steps of North America * * *."

In the voluminous discussion of this provocative paper, Lt.-Col. F. C. Temple stated (10*a*): "* * * engineers who have to deal with sewage or town refuse are not doing their job properly until they get the products from both back on to the land."

[5] Senior San. Engr., State Dept. of Health, Hartford, Conn.

In a written discussion of Sir Howard's paper, Arthur J. Martin, past-president of the Institute, stated (10*b*):

> "The Author's protest against the neglect of our domestic wastes is very timely. The Sanitarian is notoriously a wasteful animal. He delights in destruction. He flouts the wise economy of nature and disregards the food cycle which she has established. We draw heavily year by year on the fertility of our soil and we squander plant food to the tune of scores of millions of pounds. We seek to redress the balance by means of artificial manures * * *.
>
> "The persistent neglect of sewage sludge is deplorable. It is doubtless attributable in part to the intense conservatism of the average farmer, but it is largely the result of the habit, to which the Author refers, of rating the value of a manure in terms of nitrogen, phosphorus and potassium and overlooking the humus of which our fields are so sadly in need * * *.
>
> "An engineer from Czechoslovakia told me last year that the whole of the sludge from Prague, a city of a million inhabitants, is utilized year by year on an area of eighty square miles. Some of the sludge is carried to a distance of fifty miles, and the supply cannot keep pace with the demand. Such instances go to show that sludge has only to be known to be appreciated * * *."

In view of the rather detailed information available on the use of sewage sludge as fertilizer (see bibliographical references (11) through (17), it would seem that for the purposes of this Symposium this paper can be confined to summarizing the latest information on the subject, together with the writer's personal experience with the use of sludges, particularly primary digested sludge, as fertilizer, or in a more appropriate term, as a soil conditioner.

Use of Liquid Sludge Versus Dewatered Sludge

Liquid sludge normally contains from 85% to 98% water and is infrequently used as a fertilizer either in raw or digested form. The principal objections to its use are:

1. The large quantity of fluid bulk that must be transported unless pipe lines can be laid to near-by agricultural fields
2. Difficulties of uniform application to the area to be fertilized;
3. The greater likelihood of objectionable odor; and
4. The possibility of pathogenic organisms being present. C. P. Mom and C. O. Schaeffer (18) isolated typhoid bacteria in liquid Imhoff tank sludge drawn to drying beds, but after drying no typhoid bacteria could be found. Although liquid sludge does contain additional fertilizer constituents which are lost in the drained liquor by dewatering processes, past experience encourages dewatering and storage of sludge as additional safeguards against the transmission of infection. No cases of sickness have been traceable to the use of properly conditioned digested or activated sludge.

Sludge is generally utilized, after it has been partly dewatered, by draining it on beds of sand (Fig. 4), by pressing, by centrifuging, or by vacuum filtration (Fig. 5). The partly dried sludge is then referred to as "sludge cake," and ordinarily contains 70% or less of moisture. Such cake is dry enough to be shoveled and spread like manure or moist soil.

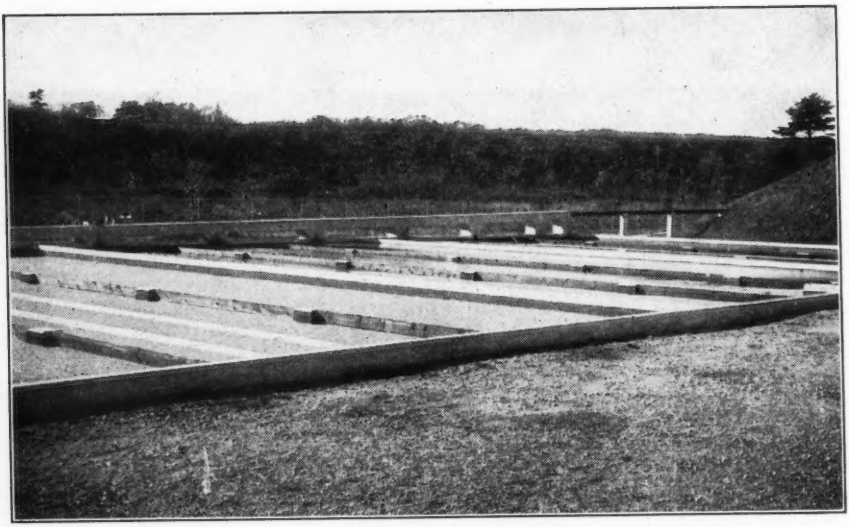

FIG. 4.—OPEN SLUDGE-DRYING BEDS

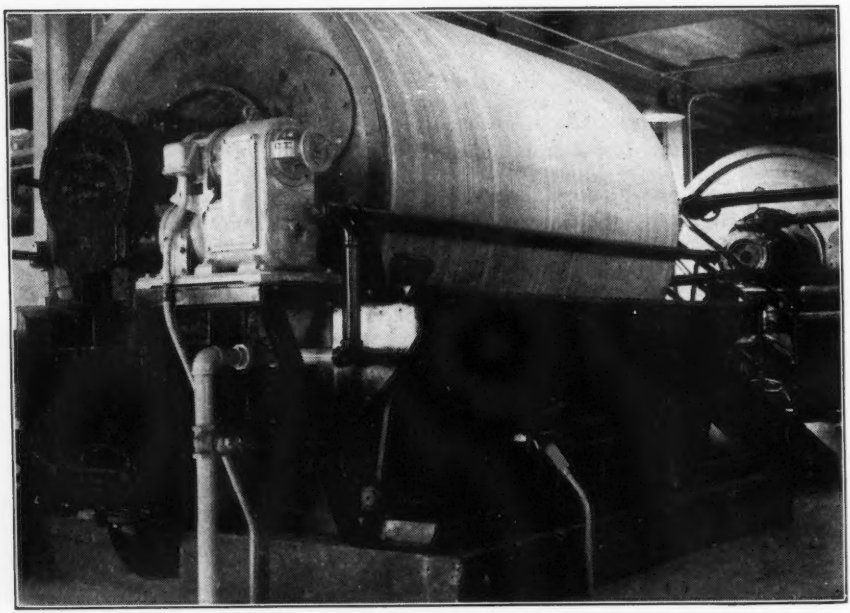

FIG. 5.—MODERN VACUUM FILTER FOR SLUDGE DEWATERING

Use of Raw Sludge as Fertilizer

The use of raw primary sludge as fertilizer is not recommended, because:

1. It is odorous;
2. It has a higher grease content than digested sludge and is not readily assimilated by the soil;
3. It tends to bring about early and persistent soil acidity, especially if the sludge has been conditioned with iron salts;
4. Industrial wastes in the sludge may poison the soil;
5. It is a potential carrier of disease unless highly limed in sludge conditioning, when the high pH (usually ranging from 10.3 to 10.6) may destroy large numbers of bacteria. If heat dried, the bacterial content may be reduced; and
6. A sixth objection is that it is not humified for soil use. In this respect, Sir Howard of England states (19):

> "The utilization of vegetable and animal wastes in crop production involves two definite steps, (1) the formation of humus and its incorporation into the soil, and (2) the slow oxidation of this product accompanied by the production of available nitrogen. Both of these stages are brought about by micro-organisms for which suitable environmental conditions are essential. The requirements of the first phase—the preparation of humus and its incorporation into the soil mass—are so intense that if the process takes place in the soil itself, it is certain to interfere with the development of the crop. The soil in fact gets indigestion! This explains to a large extent why crude sewage, crude refuse, and some of the sewage and refuse derivatives have been found wanting, or even harmful, by cultivators. In some places worse has happened; but it is clear that in many the first phase has not been sufficiently prolonged to allow the second to come into operation before the stuff was offered to the crop."

Activated Sludge

Raw or heat-dried activated sludge has a much higher fertilizing value than ordinary primary or secondary sludge for two reasons: One, the higher content of ammonia, and the other, its much higher availability. When digested, much of this fertilizing value is lost. Heat-dried and ground raw activated sludge is an organic fertilizer material especially suited to lawns because of its nitrogenous content. The analysis of Milwaukee sludge is as follows (dry basis):

Constituent	%
Nitrogen	6.2–6.5
Available phosphoric acid	2.5–2.75
Potash	0.5–0.75

The drying process is generally believed to free such sludge of harmful bacteria, although it does not completely sterilize it.

Dewatered Digested Sludges

Much of the digested sludge produced in the United States is so-called primary digested sludge—that is, from sewage solids that have been settled in tanks and subjected to anaerobic digestion. Secondary or humus sludge is frequently mixed with primary sludge and digested with it. Most of this paper

is concerned with digested sludge. A comparison based on chemical analysis of digested primary sludge and related materials is afforded in Table 16. Items 2, 3, and 4, Table 16, are as reported by E. E. DeTurk (20) and Thomas P. Maloy (21).

TABLE 16.—COMPARISON OF TOTAL CHEMICAL CONTENTS (PERCENTAGE BY WEIGHT, DRY BASIS)

No.	Substance	Nitrogen	Phosphoric acid	Potash
1	Digested sludge[a]	1.7 (total)	1.5	0.15
2	Manure[b] (20)	1.97 (total)	1.29	1.96
3	Horse manure (21)	0.7 (available)	0.11	0.45
4	Sheep manure (21)	1.8 (available)	1.25	3.0
5	Commercial potato fertilizer	5.0 (total)	8.0	7.0

[a] Average of samples from a number of sewage plants in Connecticut. Some sludge cake produced in Connecticut also contains about 10% calcium oxide (lime). [b] Supposedly a mixture of animal wastes.

The nitrogen in sludge comes into action slowly, thus removing the danger of so-called burning of crops. Alleged burning of grass is usually caused by over-application of sludge cake, thus actually smothering the growing blades of grass, particularly if it is young.

Apparently sludge compares favorably (Table 16) with the ordinary run of manures. Its principal deficiency is in potash. According to DeTurk (20*a*): "The most important differences in the two materials are that the more easily decomposable nitrogen compounds of sludge have been more completely removed by its digestion process than is the case with manure, leaving a nitrogenous residue of lower availability (this is not shown by the table); and that the potash being mostly water-soluble has been largely washed out of the sludge while it is retained by the manure."

The chemical analysis of sludge does not, however, tell the entire story. Primary digested sludge is of considerable value as a soil conditioner, from its content of humus. Approximately 50% of the dry weight of sludge may be humus. Most of the organic matter in digested sludge is humus except for the ether-soluble fats. A sandy soil is particularly benefited by sludge, since its water holding capacity is increased. Unlike commercial fertilizers, sludge is not a balanced food for plants, nor is it an all-purpose plant food. Its application, however, works a mechanical change into soil, and improves its texture. The continued exclusive use of commercial fertilizers represents unsound and dangerous farming practice. Soils repeatedly planted to growing crops require the organic humus such as is found in manures, peat moss, green cover crops, and sludges. The value of sludge should not be judged only by comparison of chemical analyses with artificial fertilizers, but by the results it produces in plant growth.

Sir Howard declares (10*a*), in connection with the use of unhumified materials:

> "We seem to be always thinking in terms of soil nutrients. This is a great mistake. In agriculture it is a question of the utilization of fungous tissue mobilized in the form of humus which really matters and which is one of the secrets of the agriculture of the East. This fungous question

is far more important than nutrients and in this matter we have gone on completely wrong lines."

Rudolfs (22) believes that the soil improving and growth promoting values of sewage sludge are distinct and additional values to its fertilizer or plant food content by replenishing of organic matter and the minor elements needed by plants.

Digested sludge is comparatively free from weed seeds which are largely destroyed in the digestion process. Most manures on the other hand contain weed seeds which prove very objectionable, particularly when spread on lawns. Tomato plants and a few of the cucurbits may sprout in digested sludge, especially if the digesting temperatures are below 80° F, but they are not ordinarily troublesome.

Application of Digested Sludge and Hygienic Considerations

Sludge may be applied as a top dressing in the case of trees, shrubs, or lawns; in the hill in the case of melons, squash, or other vine crops; or it may be spread and then plowed or forked into the soil. When used as a top dressing for lawns, it should not be applied too thickly or it may choke the growing vegetation. Sludge may be applied when the ground is frozen or covered with snow. In such cases, it is raked and pulverized into the soil in the spring. Grinding sludge cake with machines (Fig. 6), or breaking it into smaller

Fig. 6.—Grinding Makes Sludge Cake Suitable for Ready Use as a Fertilizer

particles by hand, greatly enhances its application as a top dressing. Many sewage plants now wasting their sludge output would find a ready market if grinding were instituted. Except where applied to grass, trees, shrubs, or

perennial beds, sludge should be placed in the hill or plowed under in the fall or early spring before planting the crop.

Digested sludge is safe from the health standpoint if it is mixed with the soil and no more is applied to the growing crop. It is safer not to plant root crops eaten raw, such as carrot or radish, or vegetables grown in close contact with the soil, and eaten raw, such as lettuce, celery, or onions, in ground recently fertilized with digested sludge, unless the sludge cake itself has been subjected to storage for at least a month.

The digestion of sludge, combined with the more common methods of dewatering, such as vacuum filtration (in which case most sludges are conditioned with lime, giving a high enough *pH*-value to kill great numbers of bacteria) or air-drying, and its storage afford sufficient protection against intestinal diseases. However, reasonable care should be exercised in handling it.

Amount of Digested Sludge to Be Used

Although an excess of sludge will rarely do harm, except in cases of outright mechanical smothering of growing things, the following suggestions have been made for its application:

1. For orchards (15)..... 10 tons wet sludge cake per acre (or about 11 cu yd per acre)
2. For grass (16)..... 10 to 20 cu yd per acre (or 0.6 to $1\frac{1}{4}$ cu ft per 100 sq ft of soil)
3. For vegetables, shrubs, and flowers (15)..... 20 to 60 cu yd per acre (or $1\frac{1}{4}$ to $3\frac{3}{4}$ cu ft per 100 sq ft of soil)

Fortifying Digested Sludge

By mixing dried sludge with chemicals a balanced fertilizer can be obtained with the desired plant food. Fortified sludge should be applied in smaller amounts to the soil depending on the type of crop and the local conditions. The following formula is suggested for general use and is suitable for almost any crop or soil:

Chemical	Pounds
Sludge (dry basis).....	360
Ammonium sulfate.....	25
Acid phosphate (20%).....	50
Muriate of potash.....	15

In the case of dry sludge, 360 lb amounts to about 900 lb of sludge cake as removed from the average sand-drying bed, or 1,200 lb of cake as removed from the average vacuum filter. This is, roughly, two thirds of a cubic yard, or about 20 cu ft. Other formulas may be used for special cases, or for a particular soil.

Use of Sludge as Fertilizer

Sludge is particularly adapted to lawns, golf courses, pastures, and meadows, as well as to the flower and vegetable garden. Maloy (21) reports that trees fertilized with sludge had a healthier foliage, both in amount and

color, and retained their leaves for a longer time in the fall than near-by unfertilized trees. A. M. Rawn, M. Am. Soc. C. E. (23), in 1941 stated that sludge has been used as fertilizer for citrus groves for about thirteen years with convincing results.

Sludge deepens the green color of grass and stimulates a luxuriant growth. Its benefits are noted into the second year. For flower beds it provides a much-needed humus for the hot summer months as well as a moderate but long-yielding amount of nitrogen. In vegetable gardens, it is beneficial for corn, potatoes, beans, spinach, asparagus, and vine crops.

For years the writer has grown plots of vegetables, shrubs, flowers, and grass with and without sludge. Its worthiness as a soil conditioner has been demonstrated by excellent plant growth and high yields.

When used continuously on land, sludge tends to acidify the soil. Hence, corrective measures must be taken every three or four years to avoid damage to plants requiring a neutral or near neutral soil, by checking acidity (caused largely by the breakdown of the greases and fats present in sludge) through judicious applications of powdered limestone, or hydrated lime.

Economic Value of Digested Sludge

While digested primary sludge does not have the market value of raw activated sludge, it may have an estimated value (based primarily on its chemical analysis only and without regard to its valuable humus content) of at least 50¢ per cu yd for unground sludge. Ground sludge brings a better price and is generally in demand. Mr. Rawn (23) states that dairy manure in Southern California in 1941 was selling for $1.50 per ton, delivered at the roadside. This is the commercial variety containing about 1.25% nitrogen. Digested sewage sludge (roadside delivery) was selling for $7.00 per ton in the same area. The difference in selling price, according to Mr. Rawn, is not indicative of high-pressure salesmanship; to the contrary, it portrays better than any other factor the difference in value, and is excellent proof of the fertility value of the product.

At most sewage plants, sludge is given away, or used as fill. The potential promoter or user of sludge can reasonably consider the cost of manure in his community as a rough index of the value of local sludge.

Why Digested Sludge Has Not Been More Widely Used as a Soil Conditioner

What are some of the reasons why digested sludge has not been used more extensively as a soil conditioner? Undoubtedly an esthetic dislike to its use, because of its source, and the fear of producing offensive odors are paramount in the minds of many laymen and agriculturists. Since properly digested, dried, and stored primary sludge and heat-dried and ground, raw activated sludge have been established as safe hygienically for application to the soil, there appears little room for argument from the health standpoint. With regard to odor, many of the more common animal manures, including cow, pig, and sheep wastes, have as much or more odor than sludge, especially if applied before thoroughly rotted or composted.

Another reason why sludge is not used is the lack of promotion for its use by many sewage plant superintendents and supervising officials. In some cases this is no fault of the operating personnel but is caused by a lack of time. The primary job of sewage treatment plant personnel is the protection of the public health and the prevention of nuisance from raw sewage, and not the manufacture or sale of fertilizer. Sludge disposal as fertilizer, when it involves salesmanship, storage facilities, grinding equipment, and in some cases bagging facilities, delivery and bookkeeping, is beyond the scope of many of the smaller treatment plants.

An objection to the use of sludge as a fertilizer is the lumpy or chunky physical form of much sludge cake, which makes it unsatisfactory for lawns, shrubs, and flower beds. Heat-drying and grinding, or grinding alone, produces a product much more suitable for home use.

There is a dearth of information among laymen on the proper application of sludge; its limitations; the need of occasional liming, and the types of vegetation for which it is adapted. Many homeowners and market gardeners could make excellent use of local sludge, but do not appreciate its humus value.

Because of the large quantity of sludge to be disposed of, or transportation costs, or other factors, the disposal of sludge cake by means other than as fertilizer has been found more practical or economical in most sewage treatment plants located in industrial areas or on the seaboard. Demands for sludge as fertilizer are seasonal. Storage is expensive. The cost of purchase and operation of equipment for drying and grinding wet sludge cake is appreciable. The cost of bags and bagging machinery is considerable. There is also the question of the market locally available. All these factors must be weighed in considering the problem.

In spite of these acknowledged limitations, however, the 1940 sewage plant operation reports for the cities of Minneapolis and St. Paul (Minn.) and New York, N. Y., carry favorable comments on the possibilities of using sludge at these plants for fertilizer rather than to continue present disposal methods of incineration and barging to sea.

According to Mr. Schroepfer (24), author of a companion paper in this Symposium:

> "When the Twin Cities sewage treatment plant was designed it was realized that an assured practical and economical method must be installed for regular day in and day out disposal of the large quantities of raw sludge filter cake to be produced. Incineration offered and still affords the most practical and economical method of disposal.
>
> "However, the Sanitary District has realized from the start that the sludge cake would have a certain value as a soil conditioner or low-grade fertilizer, somewhat comparable to manure, and that eventual development of its use for such a purpose would be desirable and might be of economic advantage to the District as well as to the farmers and people of the State. Manure is becoming scarce and a real need exists for a cheap, humus-containing fertilizer. The present method of disposal by incineration seems an economic waste of a natural resource, but the Sanitary District has no alternative unless another method can be developed at no added cost to the people of the Twin Cities. It is the District's hope that at least a portion of the sludge cake may be disposed of for use as a soil conditioner, to everyone's mutual benefit."

The report continues with remarks on experiments conducted to date. Results, it is reported, appear favorable, and it is noted that some of this raw sludge has been used in a fresh form. However, it is being recommended to experimental users that the sludge be stock-piled or composted for one month or longer before use. Not only will composting render it more friable and therefore easier to apply but, as has been discussed, it will permit changes to occur in the fresh organic matter so that a humus material for plant growth is available. Precautions from the health standpoint are mentioned, and it would seem that this phase should receive study before it is used extensively.

The New York City Department of Public Works states in its 1940 report (25):

> "The two primary objectives of sewage treatment are first, to reduce the sewage wastes to a liquid sufficiently pure to be unobjectionable, harmless, and relatively pure and secondly, to dispose of the impurities removed as sludge in an economical and unobjectionable manner. In this latter respect, this department has not lost sight of the possible value of the by-products of sewage treatment. In addition to the value of the combustible methane gas produced by the digestion of sewage sludge, the sludge residue after digestion has a distinct value as a soil conditioner."

The larger plants, of practical necessity, must grind and dry the sludge cake to create a sufficient market; otherwise the sludge is not in satisfactory physical form for immediate use and the high moisture content (60% to 70% in most cases) reduces materially the weight of actual dry sewage solids per pound of material. However, there are numerous small sewage treatment plants scattered over the United States from which natural digested sludge cake is obtainable for the cost of hauling. Certainly, such material is worth reasonable hauls of five to ten miles.

Conclusion

Admittedly, there are problems to be overcome at many treatment plants before all or (in some cases) even a part of the sludge cake can be disposed of as a soil conditioner. However, in view of the dearth of farm manure, the outlook for such disposal is encouraging with sound development.

EXPERIENCE OF CHICAGO, ILL., IN THE PREPARATION OF FERTILIZER

BY WILLIAM A. DUNDAS,[6] ESQ., AND C. P. MCLAUGHLIN,[7] ESQ.

SYNOPSIS

In the sludge disposal of The Sanitary District of Chicago (Ill.) a method has been worked out by a chain of experiments to dewater and heat dry activated sludge for use as fertilizer material or for incineration. The principles of design are indicated from the heat standpoint together with the control of odors. A typical heat balance is presented, with a discussion of the relative merits of various fuels. At the Southwest Works a combination of power generation with heat drying has been successfully used.

GENERAL

The Sanitary District of Chicago was established in 1890 to divert the sewage from Lake Michigan and later to treat sewage from a present area of approximately 442 sq miles. The present sewage load is from a population of approximately 4,000,000 and an industrial waste load that is equivalent to an additional population of nearly 3,000,000, making a total population equivalent to 7,000,000.

Practically all of the sewage is treated in four main plants (see Fig. 7)—the North Side, Southwest, West Side, and Calumet Sewage Treatment works. All plants, with the exception of the West Side Works, utilize the activated sludge process. Solids tributary to the West Side Works are disposed of by digestion in Imhoff tanks and drying on underdrained sand beds. Waste activated sludge is disposed of by dewatering, heat-drying, and processing for fertilizer uses or incineration at the Southwest and Calumet works. The waste activated sludge from the North Side Works is pumped to the Southwest Works for disposal with its waste sludge.

The sludge disposal facilities of the Calumet Works are designed to dispose of a daily average of 40 tons of dry solids by dewatering to 80% moisture on vacuum filters and heat-drying for fertilizer uses or incineration. The plant went into operation in July, 1936.

The sludge disposal facilities of the Southwest Works were designed to dispose of an initial average of 375 tons of dry sewage solids per day (100 tons of which comes daily from the North Side Works) by dewatering to 80% moisture on vacuum filters, followed by heat-drying for fertilizer uses or incineration. This plant went into operation in June, 1939.

When construction of the North Side Works was begun in 1923, sludge was to be disposed of by digestion in the Imhoff tanks of the West Side Works,

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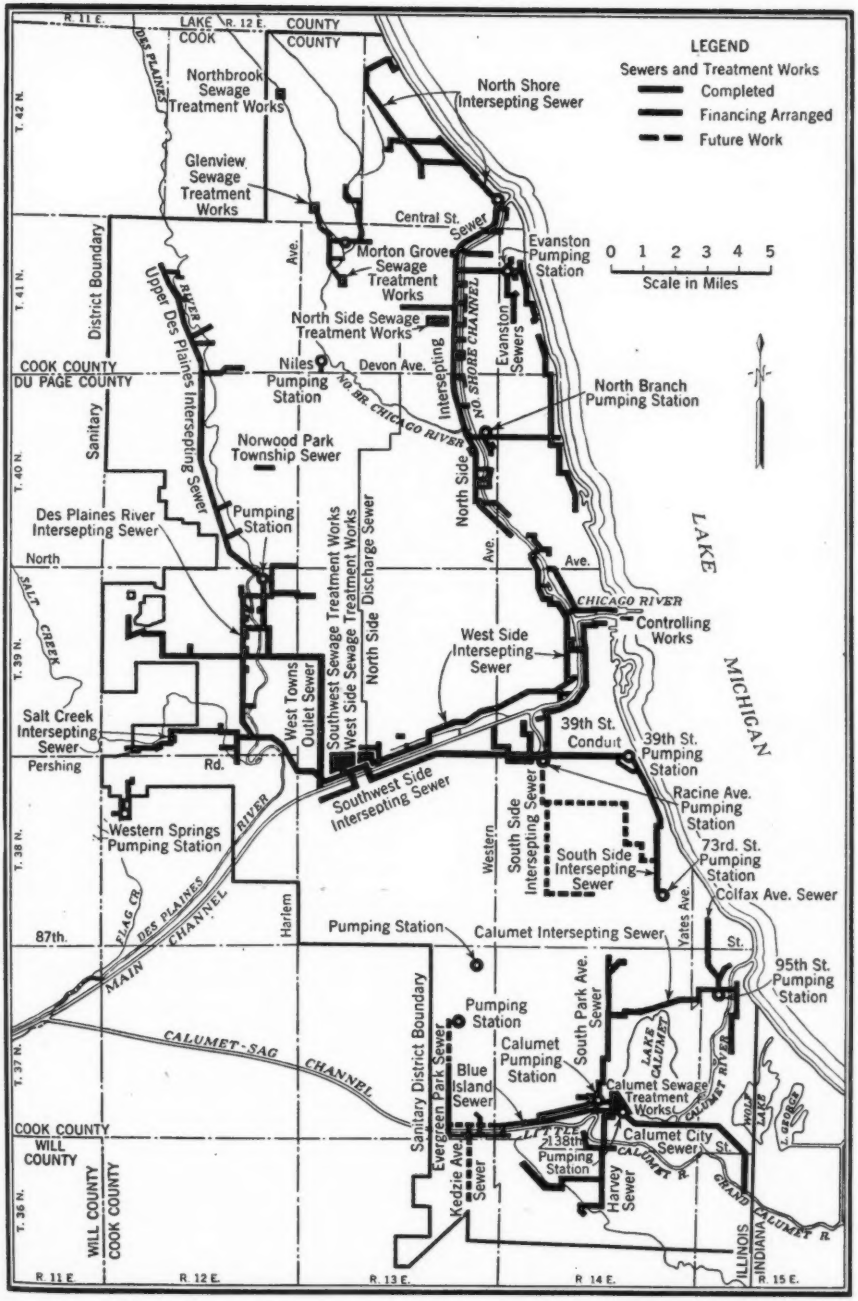


FIG. 7.—SEWAGE TREATMENT PLANTS, SANITARY DISTRICT OF CHICAGO, ILL.

together with fresh sewage solids. The waste activated sludge was therefore pumped to the West Side Works. Although such mixtures of fresh sewage solids and activated sludge can be digested in this manner, the high moisture content of the digesting mixture indicated that a large tank capacity would be required. Moreover, the drying of such large volumes of digested sludge on underdrained beds would require large additional areas of land, and the use of the beds would be restricted to the warmer months of the year.

An intensive study of mechanical dewatering, heat-drying, and incineration was undertaken in 1931. An experimental plant handling 20 tons of dry solids daily by dewatering, drying, and incineration was constructed at the West Side Works and placed in operation in the summer of 1932. By February, 1934, this plant was remodeled to develop the flash system as applied to the drying of sewage sludge and to incineration by burning in suspension. Each of these plants was operated for a year on various sludges and on sludge mixtures. Throughout these tests a continuous type vacuum filter with an area of 570 sq ft was used for dewatering. Other tests were conducted on dewatering with a bag type of filter press and a paper-making machine.

The discovery by John R. Palmer, in 1925, of the efficacy of ferric chloride as a sludge conditioner was upheld by tests of other reagents. From this exhaustive group of experiments and from experience at the pilot plant at Calumet, certain fundamental facts and principles stood out as requisite for the efficient dewatering, drying, and combustion of sludge.

In the processing of activated sludge for fertilizer or burning, its moisture content must be reduced from approximately 97% to between 5% and 10%. To produce 1 ton of sludge containing 5% moisture, from sludge containing 97% moisture, requires the removal of 64,561 lb of water. Such a quantity of water is best removed by a combination of filtration and heat-drying. Economic and practical operating considerations have established the continuous type vacuum filter as the most satisfactory mechanical dewatering equipment for sewage sludges.

Before conditioning and filtration, sludge should be concentrated to reduce the quantity of water to be removed by the filter and to provide a filter feed of uniform consistency, which is a vital factor in successful filter operation. Sludge as drawn directly from final settling tanks or digesters may vary greatly in consistency. It is recommended, therefore, that some form of concentration be provided in all instances to assure uniformity in filter feeds.

Most sewage sludges do not filter freely and therefore require the addition of a conditioning chemical which coagulates the solids and releases the water. In dewatering sewage sludge by vacuum filtration, some form of conditioning has always been necessary. Ferric chloride alone is probably the most effective and economical reagent available for conditioning activated sludge. With other types of sludges, combinations of ferric chloride and lime are used and in some instances lime alone. Other available but probably less effective reagents are chlorinated copperas and ferric sulfate.

Another paper in this Symposium indicates that elutriation of digested sludge seems to offer a successful means for reducing the amount of chemical required for conditioning and improvement in filter operation. Elutriation or

washing of the sludge seems to effect the removal of certain compounds in digested sludge that consume conditioning chemicals without producing conditioning effects.

Ferric chloride is the most expensive item of filtration cost in handling activated sludge. Close control of dosing is required. Underdosed or overdosed sludge will result in high cake moistures and low yields. The amount of ferric chloride required will be dependent upon the characteristics of the sludge involved. In Chicago the amount of ferric chloride varies daily, with a seasonal swing. Actual operating experience only will demonstrate the requirements for a particular case.

In the layout of the Chicago works, devices are installed to proportion the ferric chloride accurately and to mix it thoroughly with the sludge. Most devices that are available for the performance of this important function, although theoretically correct, fail in practical operation due to their complexity and inability to withstand the action of the chemical. Filtration should take place as quickly as possible after dosing and mixing. Continuous conditioning is used in Chicago. Routine filter operation requires checking at frequent intervals to determine the amount of chemical required for best operating results. Ferric chloride requirements of The Sanitary District of Chicago for conditioning activated sludge vary between 4% and 7% by weight of dry solids in the cake. Under these conditions, cake moistures may vary between 78% and 85%. Indications are that the amount of ferric chloride required may be related to the volatile content of the sludge.

On a continuous vacuum filter, as the cake mass forms on the filter cloth, each particle is under the pressure of the surrounding liquid due to the vacuum behind the cloth. As a particle lodges in the face of the cake, its motion decreases and the water flows by. A frictional drag is produced that causes the individual particles massed in the cake to press against each other. The finer particles will be carried on with the liquid into the cake to the point where they are stopped by frictional resistance either in the cake or within the filter medium. Examination of cake removed from a filter will show that it is more dense on the side next to the cloth than on the side away from the cloth. As the density increases, the size of the capillaries decreases and the resistance to flow of water increases, thereby increasing the time rate of filtration. In practical filter operation, when the density increases to such an extent that the passage of air and water ceases, the cake in some cases will crack and open at the weaker points in its structure. When this occurs, the pressure on the cake is relieved and further extraction of water is impossible.

The filtration rate for a compressible substance like activated sludge does not increase in proportion to the pressure because the increased pressure decreases the size of the voids between the particles, thereby restricting the flow of water. Although this phenomenon may not be the complete explanation for the difficulties encountered in the removal of water from activated and similar sludges, it is well substantiated in part by operating results in Chicago.

Filter operating costs at The Sanitary District of Chicago per ton of dry solids on a percentage basis are as follows:

Item	Calumet	Southwest
Ferric chloride	40	55
Labor	21	23
Power	8	8
Filter cloths and wire, etc.	2	3
Maintenance	29	11

At Chicago, ferric chloride is purchased in solution on a semi-annual contract basis. The last contract in 1941 was at the rate of 1.255¢ per lb of anhydrous ferric chloride, with freight on water allowed, making a cost per pound at the Chicago works of approximately 1.54¢.

The life of the filtering medium is largely a function of time rather than tonnage output. Cloths are subject to rotting, and their life is frequently determined by a crust of iron oxide formed on the back of the cloth. At Chicago, with ferric chloride conditioning of activated sludge, a long-nap woolen blanket has proved most satisfactory and economical. The life of a woolen blanket is approximately 3½ months. The cost per square foot, based on recent bids, is 18¢. The life of winding wire is approximately the same as the life of the filter medium. No attempt is made to reuse this wire. The use of woven wire for cloth backing has not been satisfactory.

Wood backing is being tried on a limited number of filters in Chicago, and indications are that it may prove more satisfactory and economical than wire cloth.

Heat-Drying

After sludge has been dewatered as far as economically possible by mechanical means, it must be dried thermally before it can be burned or processed for fertilizer. This drying process consists of transferring sufficient heat to the sludge to evaporate the entrained moisture. It is necessary to heat the sludge to the saturation temperature corresponding to the pressure in the system and to supply the latent heat of vaporization to the moisture. Since the evaporated water always leaves the dryer at a temperature above the saturation temperature, it is superheated and this requires additional heat.

In experimental work of The Sanitary District of Chicago, it was found desirable to separate the functions of drying and combustion so that each one would progress independently of the other in a separate unit especially designed for the efficient performance of its particular function.

The fundamental principles and factors found necessary for economical and efficient drying, free from nuisances, to prepare sludge for efficient combustion or for use as a fertilizer are as follows:

(1) Maximum surface exposure;
(2) Thorough agitation of the wet material;
(3) Temperature potential of gas;
(4) Velocity of drying gas;
(5) Responsive and flexible temperature control;
(6) Deodorization of gases vented from the drying operation;
(7) Instantaneous water removal; and
(8) Preservation of fertilizer qualities.

All drying operations utilize the first four of these principles in a more or less efficient manner. All eight of them are efficiently performed in the flash-drying equipment in use at Chicago.

Flash drying is a continuous operation in which the sludge in a finely divided state is dried instantaneously in a hot gas stream. At Chicago a mixture of superheated water vapor and other gases is used as the drying medium. The temperature of the drying medium as it enters the system is approximately 1,100° F, and the velocity is approximately 5,000 ft per min. At this temperature the evaporation of the moisture is almost instantaneous, and the passage through the drying cycle is a matter of seconds. Research by F. W. Mohlman has shown that dried sludge from a flash-drying system shows practically no trace of pathogenic organisms. The sludge never reaches a temperature sufficiently high to affect any of the properties valuable as fertilizer. The average temperature of the dried material as removed from the drying system is approximately 180° F. The temperature of the vapor and gases vented from the drying systems to the furnaces for deodorization is approximately 350° F. The drying systems are operated slightly under atmospheric pressure to prevent the escape of dust and gases. All operations are automatically controlled. Briefly, the sequence is as follows:

Activated sludge cake containing 78% to 85% moisture as discharged from the filters is conveyed to individual mixers where it is mixed with sufficient sludge, previously dried in the system, to reduce the moisture content and prepare a nonsticking friable mass. In this form the sludge is introduced into the drying system, where it is picked up in the high-velocity, high-temperature gas stream. A cage type of disintegrating mill interposed in the drying circuit breaks the sludge into small particles, causing maximum exposure of wet surface and complete dispersion of the particles in the drying medium. The sludge particles carried in the superheated vapor and gaseous drying medium are discharged into a cyclone separator where dried sludge is separated from the vapors and gases in which it was conveyed. Part of the sludge removed from the separator is returned to the sludge mixer, and the surplus is withdrawn as fertilizer material or pneumatically conveyed to a separate furnace where it is burned in suspension. When fertilizer is being produced, the heat required for the drying system is furnished by the combustion of auxiliary fuel. A portion of the vapors and gases vented from the sludge separators is recirculated through a heat exchanger where it is reheated by an exchange of heat from the hot products of combustion vented from the furnace and returned to the drying system. The surplus or waste vapor is vented to the furnace for deodorization.

Heat for sludge drying may be carried directly by furnace gases or it may be transferred from furnace gases to air or water vapor by means of heat exchangers. Air-drying has two distinct disadvantages. It results in lower efficiency since it is an additional medium introduced to the system, which must be raised from room temperature to the stack temperature, and it supplies oxygen which might cause fires.

The rate of evaporation is dependent on the temperature of the drying medium, the size of the sludge particles, and the rate of removal of the water

vapor. The greater the temperature differential between the drying medium and the sludge, the greater the evaporation will be. The sludge particles should be kept as small as possible during the drying process, since the smaller they are, the greater will be the area of wetted surface exposed to the heat. In other words, more moisture is in direct contact with the hot gases. Water vapor formed in the center of a particle of sludge must force its way to the surface to be liberated, and thus the smaller the sludge particles, the shorter will be the distance traveled by the vapor. When the particles are too large, the surface may become dried and case-hardened before the moisture in the

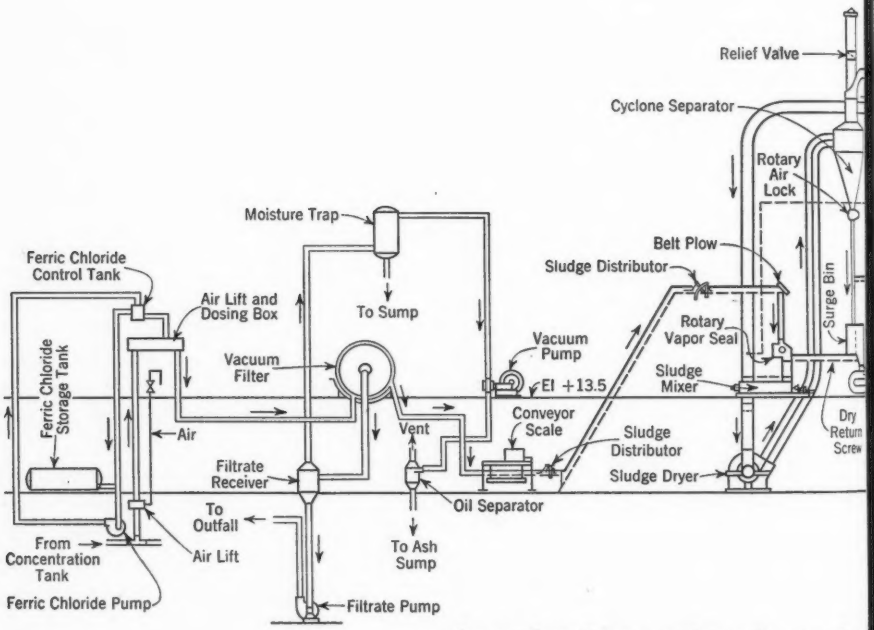
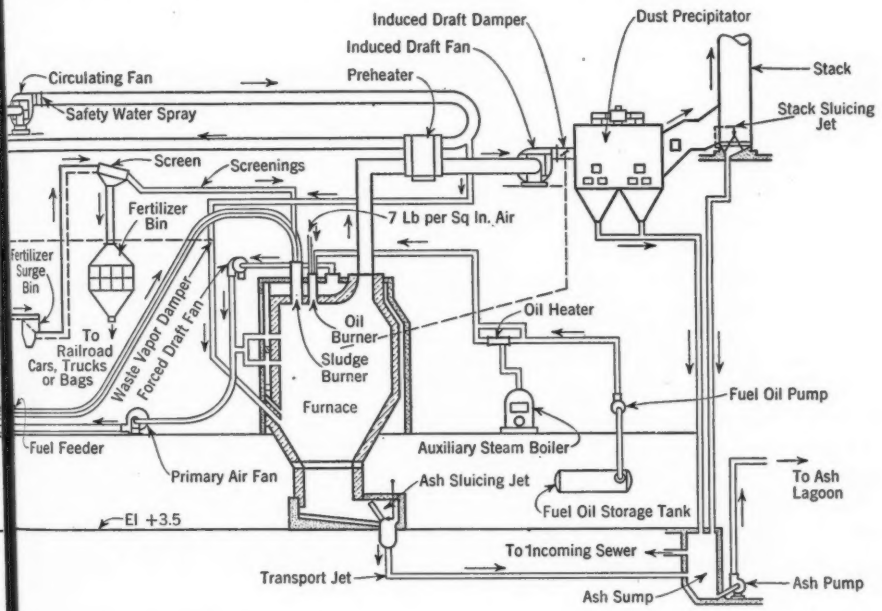


FIG. 8.—FLOW DIAGRAM OF SLUDGE DISPOSAL PLANT

center has been evaporated. This hard surface offers resistance to the release of the interior moisture and results in the dryer discharging a product that is dried on the surface but has a wet center. The rate of drying is also affected by the relative velocity between the material and the drying medium. When vapor is released from the sludge, it tends to form an insulating medium around the sludge particle, retarding further evaporation unless it is instantly removed. Failure to remove the water vapor results in an increased pressure in the system that requires more heat to evaporate the water in the sludge.

Responsive and flexible control is a vital necessity in any drying and combustion operation. Where sludge is dried for incineration, it is desirable that the character of the material and its moisture content as fired be maintained as uniform as possible. Where sludge is dried for fertilizer uses, it is necessary

that the size of particles be kept uniform and that the temperature of the drying system be automatically controlled to avoid driving off valuable fertilizer ingredients, and to insure a moisture content in the dried material between 5% and 10%. Where drying and combustion occur independently of each other, automatic temperature control is a relatively simple matter, inasmuch as the demand for heat in the drying system can be met by liberating more or less heat in the furnace. With the cage type of drying mill used by the Sanitary District, the size of particle is dependent upon the speed of the mill, its interior arrangement, and character of the sludge feed.



EXTENSION OF CALUMET SEWAGE TREATMENT WORKS

Sludge, like other similar organic materials, has an inherent tendency to heat and burn spontaneously when stored in large quantities over a period of time. This tendency may be caused by bacterial or chemical action or a combination of both. It is also a function of the moisture content, fineness, and initial temperature of the material stored. Experience at Chicago indicates that the moisture content should be held below 10%, the heat-dried sludge should be as free as possible from dust, and the temperature at the time of loading in cars should be below 100° F. This is accomplished by screening, dedusting, and cooling in water-jacketed conveyers.

All heat-dried activated sludge as produced for fertilizer uses at Chicago is loaded in cars in bulk, after screening and cooling, for immediate shipment to consumers. No material at present is bagged or stored in bulk. Facilities for

a limited tonnage of bulk storage are now under construction at the Southwest Works.

Fig. 8 is a schematic diagram of the process at the Calumet plant. Three drying systems of the flash type, with individual furnaces, are installed at this plant. Each is designed to evaporate 6,500 lb of water per hour and to handle 20 tons of dry solids per day.

Fig. 9 is a schematic diagram of the Southwest plant. Eight drying units are installed at this plant. Each unit has an evaporation capacity of 20,000 lb of water per hour and handles approximately 62.5 tons of dry solids per day.

Combustion

In the combustion of sewage sludge as well as commercial fuels many factors are involved, due to the great variation in their characteristics. Specific requirements for the proper combustion of a particular sludge must be considered as a distinct problem requiring individual consideration. However, it is possible to draw certain general conclusions as to the combustion requirements of sludges. These conclusions form the basis of design of all combustion apparatus and thus are worthy of careful study.

The term "combustion," as applied to furnaces, is used in referring to the vigorous reactions of oxygen with different substances, whereby heat and light are liberated. Combustion therefore differs from other slower reactions with oxygen only in the vigor of the reactions and in the forms and quantities in which energy is liberated.

The chemistry of combustion of sewage sludge is elementary and differs but slightly from that of other solid fuels. The elementary substances encountered in the combustion of sewage sludge and commercial fuels are oxygen, nitrogen, hydrogen, carbon, sulfur, and combinations of these elements (see Table 17). Oxygen is the sole supporter of combustion. A combustible substance is one capable of combining with oxygen to produce heat. Combustion is complete when the combustible elements and compounds have united with all the oxygen they can use.

In the combustion of sludge, carbon and hydrogen are the most important of the combustible elements from the standpoint of heat liberation. These elements occur either in a free or combined state as in all solid, liquid, and gaseous fuels. For the most part, the hydrogen, carbon, and sulfur compounds in sludge are unstable in nature. In sludge cake, these compounds are associated with a relatively high percentage of water and ash, as compared with commercial fuels.

Every combustible substance has a definite ignition temperature to which it must be raised before it will combine chemically with oxygen. Below this temperature no combination takes place. Furthermore, this ignition temperature must exist in the presence of sufficient oxygen or there will be no combustion. The ignition temperatures of combustible elements and compounds commonly found in sludges are given in Table 17(*a*), Col. 5.

In the drying and burning of sludge, odors are produced largely by the liberation of hydrocarbon compounds. By observation, this liberation is found to begin at relatively low temperatures and to increase in intensity with

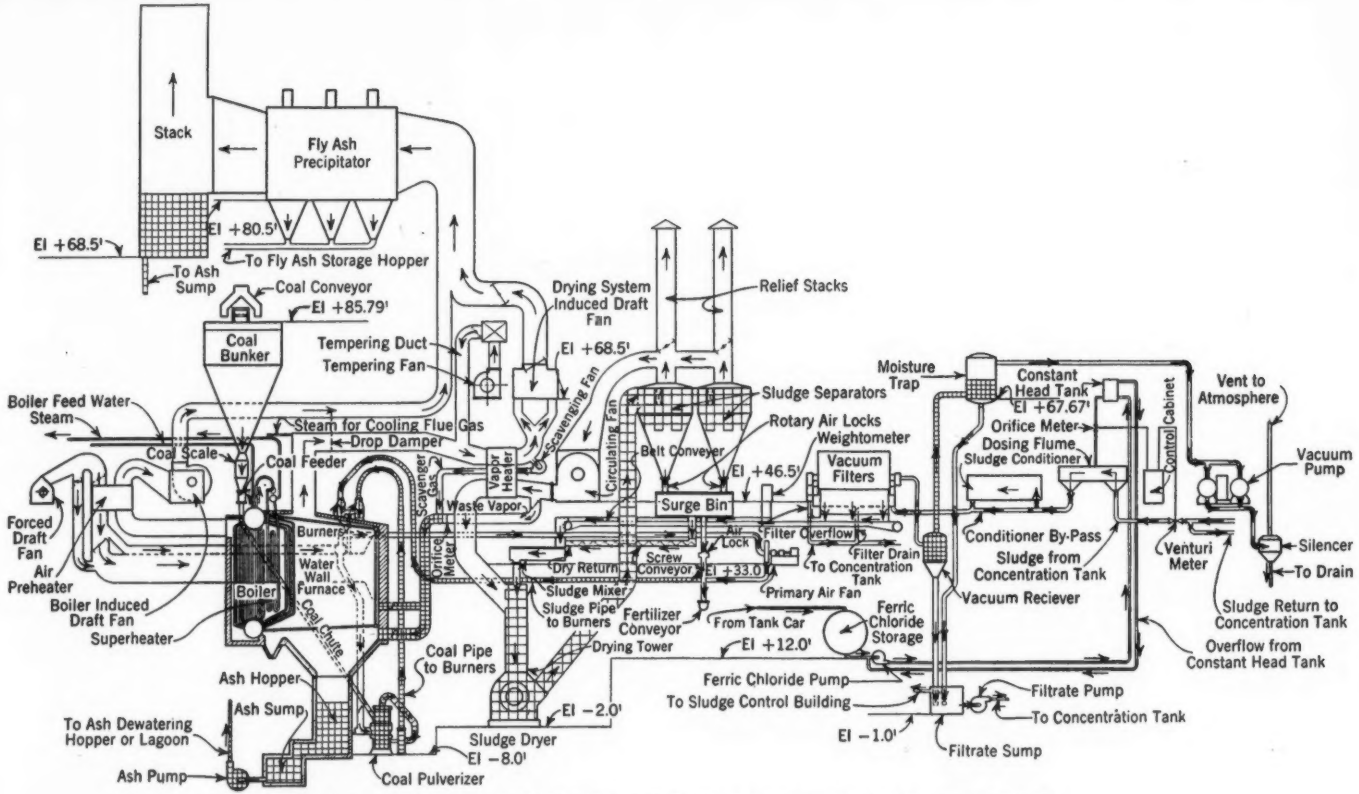


FIG. 9.—FLOW DIAGRAM OF SLUDGE DISPOSAL PLANT, SOUTHWEST SEWAGE TREATMENT WORKS

increased temperature. Odors are also released from the solids and gases in solution with the water, which is evaporated into steam. Emulsified organic fats likewise give off offensive odors with a slight application of heat. The

TABLE 17.—CHARACTERISTICS OF CHEMICAL ELEMENTS IN SLUDGE

No.	Substance	Symbol	Atomic weight	Molecular weight[a]	Ignition temperature (degrees F)	Proportion by weight (%)
	(1)	(2)	(3)	(4)	(5)	(6)
	(*a*) COMBUSTIBLE ELEMENTS[b]					
1	Carbon	C	12	12	900	35.50
2	Hydrogen	H_2	1	2	1,130	5.13
3	Sulfur	S_2	32	64	470	0.85
4	Carbon monoxide	CO	..	28	1,210	....
5	Methane	CH_4	..	16	1,202	....
6	Hydrogen sulfide	H_2S	..	34	....	....
	(*b*) OTHER CONSTITUENTS					
7	Nitrogen	N_2	14	28	....	4.61
8	Oxygen	O_2	16	..	....	17.85
9	Chlorides	..	..	..	....	0.57
10	Ash	..	..	..	....	35.49
11	Air	..	..	29	....	....
12	Carbon dioxide	CO_2	..	44	....	....
13	Water vapor	H_2O	..	18	....	....
14	Sulfur dioxide	SO_2	..	64	....	....

[a] Approximate. [b] The potential heat content of sewage sludge is 7,000 Btu (British thermal units) per lb.

ignition temperatures of the CH compounds lie between 1,100° and 1,200° F, approximately. The end products of complete combustion of all CH compounds are water vapor and CO_2, both of which are odorless. Therefore, all gases and water vapor produced in the heat-drying of sludge should be raised to a temperature of not less than 1,300° F in the presence of sufficient oxygen to effect complete combustion of the offensive CH compounds.

Likewise, sludge and similar organic materials must be burned at high temperatures with sufficient oxygen present to complete combustion if odors are to be avoided in the products of combustion discharged to the atmosphere. Sulfur dioxide, one of the end products of complete combustion of sulfur and its compounds, is objectionable if discharged into the atmosphere in large quantities. The percentage of sulfur and its combustible compounds as found in most sludges is relatively low compared with the content of many bituminous coals. Therefore, it is of little consequence. The amino compounds are dissociated at these temperatures and likewise are of little consequence.

The ignition temperature of combustible substances (Table 17(*a*), Col. 5) is well above the boiling point of water (212° F at 14.7 lb per sq in. absolute pressure). Thus, the material is thoroughly dry when ignition and combustion take place. A large ball of sludge with a hard dry surface may become ignited and smolder while the moisture is being driven off from the interior. Such combustion is inefficient and may produce odors unless sufficient auxiliary fuel

is burned within the furnace to maintain a temperature sufficiently high to ignite the combustible gases emitted by the smoldering material and dissociate any noncombustible compound of an offensive nature. The experience of The Sanitary District of Chicago has shown that complete deodorization of gases can be assured only after the gases have been raised to a temperature of more than 1,300° F in the presence of sufficient oxygen to complete combustion of the volatile compounds. Efficient combustion of sludge, free from objectionable odors, therefore, is possible only with thoroughly dried material unless the obnoxious gases are deodorized after combustion.

The chemical reaction of combustion results from a rearrangement of the atoms of the constituent elements into new compounds. It is in accordance with fixed laws and may be computed from the molecular weights given in Table 17.

Air is the source of oxygen supply for commercial combustion operations, and is considered as a mechanical mixture of oxygen, nitrogen, carbon dioxide, water vapor, argon, and other inert gases. For combustion purposes, the carbon monoxide and the inert gases are usually included as nitrogen. The generally accepted percentages for atmospheric air are:

For oxygen (O_2)—

by volume,	20.91
by weight,	23.15

and for nitrogen (N_2)—

by volume,	79.09
by weight,	76.85

The following reactions occur only when the exact amount of oxygen required for combustion is present and the oxygen is wholly utilized:

Combustible	Reaction
Carbon (to CO)	$2\,C + O_2 = 2\,CO$
Carbon (to CO_2)	$2\,C + 2\,O_2 = 2\,CO_2$
Carbon monoxide	$2\,CO + O_2 = 2\,CO_2$
Hydrogen	$2\,H_2 + O_2 = 2\,H_2O$
Sulfur (to SO_2)	$S + O_2 = SO_2$
Sulfur (to SO_3)	$2\,S + 3\,O_2 = 2\,SO_3$
Methane	$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$
Hydrogen sulfide	$2\,H_2S + 3\,O_2 = 2\,H_2O + 2\,SO_2$

Carbon enters into combustion in two forms, CO and CO_2. The former is incomplete, the latter complete. The combustion of hydrogen is always complete to H_2O. If insufficient air is supplied, a portion of the hydrogen combines with the supply of oxygen present to form water, and the remainder passes to the stack.

From the proportionate parts by weight of oxygen and nitrogen in atmospheric air, 4.32 lb of air are required for each pound of oxygen. This air also supplies 3.32 lb of nitrogen that does not enter into combustion. The ratio 3.32 : 1 is known as the "nitrogen ratio." Nitrogen serves no useful purpose and is a direct loss in combustion work, since it must be raised to the furnace exit temperature. Based upon approximate molecular weights, carbon, hy-

drogen, and sulfur require the following weights of oxygen for complete combustion:

One pound of:	has the following oxygen requirement (lb):
Carbon	$\frac{O_2}{C} = \frac{32}{12} = 2.667$
Hydrogen	$\frac{O}{H_2} = \frac{16}{2} = 8$
Sulfur	$\frac{O_2}{S} = \frac{32}{32} = 1$

The heat value of any sludge may be determined by actual experiment in a calorimeter. Where such apparatus is not available, approximate heat values may be calculated for certain combustible compounds from the ultimate chemical analysis, using Dulong's formula:

$$q = 14{,}600\ C + 62{,}000\left(H - \frac{O}{8}\right) + 4{,}050\ S \quad \ldots\ldots (1)$$

in which q is the heat of combustion in British thermal units (Btu) per pound, and C, H, O, and S, respectively, denote the proportional parts by weight of carbon, hydrogen, oxygen, and sulfur.

In the complete combustion of 1 lb each of carbon, hydrogen, and sulfur, the resulting products of combustion will be as follows:

Computation	Pounds
1 lb C + 2.667 lb O_2 =	3.667 CO_2
2.667 × 3.32 lb N_2 =	8.85 N_2
1 lb H_2 + 8 lb O_2 =	9 H_2O
8 × 3.32 lb N_2 =	26.56 N_2
1 lb S + 1 lb O_2 =	2 SO_2
1 × 3.32 lb N_2 =	3.32 N_2

Methane, one of the members of the CH series, is computed in a similar manner: $CH_4 = C + 4H$; or, from atomic weights, $16 = 12 + 4$. Thus, 1 lb of methane is composed of 0.75 lb C + 0.25 lb H. To burn 0.75 lb of carbon will require $0.75 \times 2.667 = 2.00$ lb O_2. To burn 0.25 lb of hydrogen will require $0.25 \times 8 = 2.00$ lb O_2. The total oxygen requirement is $2 + 2 = 4$ lb. The total air required is $4 \times 4.32 = 17.28$ lb. The products of combustion will be as follows:

Computation	Pounds
0.75 lb C + 2.00 lb O_2 =	2.75 CO_2
0.25 lb H + 2.00 lb O_2 =	2.25 H_2O
17.28 lb air × 0.7685 =	13.28 N_2

These computations, although simple, are important in combustion work, particularly where digester gas and liquid fuels are used as major or auxiliary sources of heat for drying and incineration. The theoretical dry air required

for a combustible compound, therefore, is:

$$A = 11.52\,C + 34.56\left(H - \frac{O}{8}\right) + 4.32\,S \quad \ldots\ldots\ldots\ldots (2)$$

in which A = theoretical weight of dry air required per pound, C, H, O, and S being the proportional parts by weights of the carbon, hydrogen, oxygen, and volatile sulfur in the compound. The term $\left(H - \frac{O}{8}\right)$ represents the available hydrogen and corrects for hydrogen combined with oxygen in the fuel. Part is probably in the form of moisture, or is combined with organic nitrates, and part may be present in carbonates of mineral matter found in the sludge or fuel. This assumption probably is sufficiently accurate for ordinary calculations.

A series of calculations for different fuels will show a wide variation in theoretical air requirements per pound of fuel. When expressed in terms of theoretical air requirements per 10,000 Btu, a close agreement will be found among all fuels, including sludge, peat, coal, and oil. The average value of such a comparison will indicate a theoretical air requirement of 7.5 lb per 10,000 Btu, with a maximum departure not exceeding 2%.

Excess Air.—From the chemical standpoint, to supply the proper amount of oxygen for complete combustion appears relatively simple; but this is not the case. The primary problem in combustion is to maintain the supply with its accompanying nitrogen as near to the theoretical amount required as possible. The efficiency of combustion, therefore, may be measured by the relation between the air actually supplied and that theoretically required for combustion. It is vitally important that the air supply be admitted at the proper time and in such a manner that the oxygen comes into intimate contact with the combustible substances in the sludge, which include the combustible solid particles and gases as they are distilled from the sludge.

The air supplied to a furnace for combustion is classified broadly as primary and secondary. Primary air is that required for propagation of flame, whereas secondary air is that introduced to complete combustion after the flame has been propagated.

As stated, nitrogen introduced into the furnace with the oxygen required for combustion performs no useful function in combustion. It passes through the furnace unchanged except in temperature and volume. It absorbs heat and reduces the temperature of the products of combustion. All these factors necessitate designing the furnace space, connecting flues, and ducts sufficiently large to accommodate the excess gases. Any excess oxygen supplied to the furnace results in the same loss as for nitrogen. Such oxygen is accompanied by additional nitrogen, which further accentuates the loss. On the other hand, where insufficient oxygen is supplied for complete combustion, the loss due to the presence of the nitrogen is inappreciable as compared with that due to incomplete combustion. For example, if 1 lb of carbon is burned to carbon monoxide, the heat produced is 4,380 Btu, whereas if the combustion is carried

to CO_2 the heat produced is 14,540 Btu. This factor will be discussed further in connection with furnace losses.

Smoke and Odors.—Smoke and odors are a widely discussed topic in connection with the combustion and drying of organic refuse and commercial fuels. Most of the definitions of smoke are unsatisfactory and vague.

The common conception of objection to smoke is based on its visibility. Most smoke ordinances are based on the visibility, density, and color of escaping stack gases, with little or no attention to the invisible or more objectionable compounds which are by far the worst offenders in atmospheric pollution. The amount of solids discharged from a stack is not a complete indication of its nuisance value. A stack may appear smokeless and still discharge a large volume of objectionable gases into the atmosphere. The color of smoke is usually imparted by carbon particles. The presence of a small amount of carbon is sufficient to give color to large volumes of invisible gases, which may or may not represent a nuisance or a combustion loss. A certain amount of color may also be given to gases by particles of fine ash and other mineral matter carried in suspension in the gases, neither one of which represents a combustion loss. When burning pulverized fuel, the amount of this material in escaping gases is considerable unless efficient means are provided for its interception, as the passing of all waste gases through some form of dust collector such as a settling chamber, centrifugal separator, or electrical precipitator.

The effectiveness of such means varies over a wide range and is dependent upon various factors, principally the dust density of the gases. The plain settling or expansion chamber, unless of tremendous volume with resultant low gas velocities, will remove little if any of the fine flocculent dust. A centrifugal separator, showing removal efficiencies as high as 94%, is more effective. This provides maximum removal per dollar invested. The most effective of all separating devices is a commercially patented electrical precipitator. Tests on this equipment installed in the Calumet and Southwest works in Chicago indicate a removal of 97% of suspended solids from the stack gases. The selection of dust-separating equipment should be based on the degree of separation desired. This is determined to a great extent by the remoteness of the plant from habitated areas. Investigations in connection with large power plants, where coal is burned in suspension or on stokers, indicate that a large quantity of the flocculent solids discharged from stacks is deposited at a distance varying from 5 to 15 miles from the source, unless efficient means for interception are provided.

The presence of a large volume of visible water vapor in stack gases is not objectionable from the standpoint of atmospheric pollution, if discharged at a height to permit complete dispersion.

In the early experimental work of the Sanitary District, all gases vented from the sludge dryer and furnace were passed through a vapor condenser and scrubber in an attempt to remove odors. This method of odor elimination proved ineffective and expensive in operation and maintenance. All the metallic parts (condenser, piping, and spray nozzles) were attacked by sulfurous acid and other compounds that formed when the vapors reached the dew point. The interior surfaces of the equipment became coated with a very corrosive,

tarry compound with an offensive odor. When heated, this substance gave off an offensive brown gas which ignited at a temperature of approximately 1,200° F and burned freely in the atmosphere. The vapor condenser and scrubber were found to act solely as a wet-type dust collector.

Heat of Combustion.—As previously stated, the term "combustion" is limited to the rapid chemical combination of the combustible constituents of a fuel with oxygen, resulting in the production of heat. The heat of combustion is therefore the heat resulting from the complete oxidation of such combustible elements and compounds. The amount of heat so generated is a constant for any given combination of combustible elements and compounds, irrespective of the manner in which combustion takes place so long as it is complete. The unit measure of quantity of heat is the British thermal unit, or the Btu, which is $\frac{1}{180}$th of the amount of heat required to raise the temperature of 1 lb of water from 32° to 212° F.

The heat of combustion of any fuel can be computed approximately from the ultimate analysis by use of the Dulong formula (Eq. 1). In computing heat values of sludges, great care should be exercised in applying Eq. 1, inasmuch as sufficient check data are not available to determine the precision of results computed by this formula. Accurate calorific values are obtainable only by calorimetric methods, which give the following values for common combustible materials:

Combustible	Heat value (q), in Btu per lb
Carbon to CO	4,380
Carbon to CO_2	14,540
Hydrogen	62,000
Methane	23,850
Sulfur to SO_2	4,050

Temperatures of Combustion.—The actual temperature resulting from the combustion of sludge or any other fuel is obtained most satisfactorily by means of a pyrometer. The theoretical temperature may be calculated from the formula,

$$T = \frac{q}{W\,C} + t \qquad (3)$$

in which T = final temperature of products of combustion, in degrees Fahrenheit; q = British thermal unit per pound of fuel; W = weight of products of combustion, in pounds per pound of fuel; C = mean specific heat of the products of combustion; and t = initial temperature of fuel and air supply, in degrees Fahrenheit.

Assuming the complete combustion of 1 lb of carbon with the theoretical amount of air supplied at an initial temperature of 60° F, then $T = \frac{14{,}540}{12.52 \times 0.27} + 60 = 4{,}360°$ F, approximately.

Such calculated temperature is entirely theoretical and is the temperature that would exist in the furnace if combustion were complete, if no dissociation

occurred, and if no heat were lost by radiation. Actually, some heat is radiated to the furnace walls before combustion is complete. At temperatures greater than 3,000° F, dissociation begins to occur, whereby water vapor breaks down into hydrogen and oxygen and carbon dioxide breaks down into carbon monoxide and oxygen. Just as the combining of these elements into water vapor and carbon dioxide liberates heat, so heat is absorbed when the reactions are reversed, resulting in a lower temperature. In computing heat balances, dissociation may be ignored since, after the gases are cooled during the process of heat recovery, the reactions are again reversed and the heat is again liberated. Since it is generally impossible to burn each particle of fuel in the presence of the proper amount of oxygen, incomplete combustion takes place, resulting in a lower temperature. Under actual combustion conditions there is always an excess supply of air present that also reduces the temperature.

In practice, factors may occur with different types of sludges and equipment which, for practical reasons, require that furnace temperatures be kept below the maximum attainable. Under such conditions high percentages of excess air are introduced into the furnace, resulting in reduced combustion efficiency. An important factor is the fusion temperature of the ash. For practical reasons, furnaces containing metallic parts must operate with a high percentage of excess air to maintain a safe operating temperature well below the maximum available.

Specific Heat.—Calculations involving combustion and heat transfer require the use of specific heat values. The specific heat of a substance is the number of British thermal units required to raise the temperature of 1 lb of the substance 1° F. Actually, the specific heat is not a constant for all conditions of pressure and temperature and, if accurate results are desired, a correction must be made for the actual pressure and temperature conditions that exist.

When heat is added to a gas while its volume is kept constant, the external work is zero, but if heat is added while the pressure is held constant, the volume increases and external work is performed. Therefore, the amount of heat necessary to raise 1 lb of a gas 1° F at constant pressure is greater than that required at constant volume because it requires heat to perform external work.

In heat computations dealing with furnace gases and sludge drying gases, the variations in pressure are small. The change in specific heat due to pressure changes thus can be ignored. The specific heat of a gas, however, varies greatly with the temperature. Any calculation in which extreme accuracy is desired should use the mean specific heat of the gas between the temperatures involved. For example, the specific heat of air is usually taken as 0.24, yet it varies from 0.238 at 50° F to 0.275 at 2,000° F.

Heat Balance.—In designing a drying and incineration system, it is necessary to make sufficient heat calculations to determine the quantity of fuel required, the air needed for combustion, the size of furnace required, and also gas quantities for determining fan and duct sizes. A summary of these data, showing the heat input to the system and the distribution of this heat among losses encountered and the useful work done, is known as a heat balance. The ratio of the heat utilized in the performance of useful work to the total heat supplied is the efficiency of the system.

A heat balance may be an actual balance obtained from the results of a test on the unit involved or it may be a theoretical balance. In the first case the fuel is weighed and its heat per pound obtained from calorimeter tests. The work done is accurately measured, as are all losses. In making a theoretical balance, certain assumptions must be made, such as radiation losses, stack temperatures, and the heat value of the fuel.

A heat balance for a flash-drying system as used in The Sanitary District of Chicago, in which the water vapor evaporated from the sludge is returned to the furnace for odor removal, would have for its heat input the heat released from the burning of the sludge and auxiliary fuel or from the burning of auxiliary fuel only if the dried sludge is prepared for fertilizer.

The heat output includes the heat used to evaporate the moisture in the sludge and the following losses:

(*a*) Radiation loss in the drying system. This consists of heat radiated from the surfaces of the dryer, ducts, separators, etc., and amounts to approximately 125,000 Btu per ton of dry solids.

(*b*) Loss due to moisture and hydrogen in the fuel. The hydrogen in the dried sludge forms water with the oxygen in the air. This water, together with the water in the sludge, must be raised to the boiling point, converted into steam, and then superheated to the temperature of the stack gases. The heat required to perform this work is taken from the heat generated from the combustion of the sludge and is therefore a loss.

(*c*) Loss due to incomplete combustion. Even when excess air is present, it is not possible to obtain complete combustion of the sludge. Some particles, for instance, drop unburned into the furnace ash hopper and others pass out as soot with the stack gases. These losses generally amount to approximately 4%. When producing fertilizer, using oil or gas as fuel, this loss is negligible.

(*d*) Stack losses. These losses correspond to the heat that is added to the fuel, combustion air, and waste vapor to raise them from the temperature at which they enter the furnace to the temperature at which they leave the system and enter the stack. These losses are unavoidable but, since they represent the greatest loss of heat in the system, they should be kept at a minimum by maintaining the exit temperature as low as possible, without risking danger of condensation, and by keeping the excess air at a minimum.

(*e*) Radiation and unaccounted losses. These losses allow for radiation from the furnace setting, air leakage into the drying system, and any losses not included in the other items.

The following is a heat balance for one drying unit as installed at the Calumet plant of The Sanitary District of Chicago, based on producing fertilizer and burning oil for fuel:

Assumptions.—

(1) Sludge feed, 20 tons per day at 80% moisture and 55° F.
(2) Average temperature, 50° F.
(3) Air, 30% and vapor, 70%, by volume in hot tower at 1,000° F.

(4) Temperature at outlet of separator, 300° F.
(5) Dried sludge removed from system as fertilizer.
(6) Fuel oil at 19,500 Btu per lb.
(7) Dried sludge, 180° F and 5% moisture.
(8) Drying system radiation losses, 125,000 Btu per ton of dry solids.
(9) Furnace radiation, incomplete combustion, hydrogen, and unaccounted losses, 10%.

Computations.—

Item	Computation	Quantity
(*a*) Dry solids, in lb per hr	$\frac{20 \times 2{,}000}{24}$	1,667
(*b*) Moisture, in lb per hr	$\frac{1{,}667 \times 80}{20}$	6,667
(*c*) Moisture evaporated in drying system, in lb per hr	$6{,}667 - \left(\frac{5}{95} \times 1{,}667\right) = 6{,}667 - 88$	6,579
(*d*) Vapor—Air Mixture (% by Weight):		
Air (30%)	1 molecular lb = 29 = 8.7 lb	41
Vapor (70%)	1 molecular lb = 18 = 12.6 lb	59
Total	1 molecular-lb mixture = 21.3 lb	100
(*e*) Specific heat of drying mixture	$0.41 \times 0.24 + 0.59 \times 0.47$	0.375
(*f*) Air leakage, in lb per hr	$\frac{0.41}{0.59} \times 6{,}579$	4,570
(*g*) Radiation losses from drying system, in Btu per hr (at 125,000 Btu per ton of dry solids)	$\frac{1{,}667}{2{,}000} \times 125{,}000$	104,000
(*h*) Waste vapor, in lb per hr	$6{,}579 + 4{,}570$	11,149
(*i*) Heat required by drying system, in Btu per hr	$6{,}579\,[212 - 55 + 970 + (300 - 212) \times 0.47]$ $= 1{,}670\left(0.30 + \frac{0.05}{0.95}\right)(180 - 55)$ $+ 4{,}570\,(300 - 50) \times 0.24 + 104{,}000$	8,131,500
(*j*) Drying mixture required, in lb per hr	$\frac{8{,}131{,}500}{(1{,}000 - 300)\,0.375}$	30,900
(*k*) Mixture to circulating fan, in lb per hr	$30{,}900 + 6{,}579 + 4{,}570$	42,049
(*l*) Heat required from furnace, in Btu	$8{,}131{,}500 + 11{,}149 \times 0.375 \times (475 - 300)$	8,863,500

At 20% air and 17.8 lb products of combustion per lb of oil (26):

Item	Computation	Quantity
(*m*)	Useful heat, in Btu per lb of oil $0.9 \times 19{,}500\ [(475 - 50) \times 0.25 \times 17.8]$	15,660
(*n*)	Oil required, in lb $\frac{8{,}863{,}500}{15{,}660}$	567
(*o*)	Products of combustion 567×17.8	10,080
(*p*)	Furnace temperature, in degrees Fahrenheit $\frac{567 \times 0.9 \times 19{,}500 + [11{,}149 \times (300 - 50) \times 0.375]}{(10{,}080 \times 0.25) + (11{,}149 \times 0.375)} + 50$	1,690
(*q*)	Recirculated gases to give 1,475° F entering the heater, in lb $\frac{(10{,}080 + 11{,}149)\ (1{,}690 - 1{,}475)}{1{,}475 - 475}$	4,570
(*r*)	Total heat supplied $567 \times 19{,}500$	11,050,000
(*s*)	Heat balance, in Btu:	
	Furnace radiation, incomplete combustion, hydrogen, and unaccounted losses, 10%	1,105,000
	Dryer radiation losses	104,000
	Dried sludge losses $1{,}670\left(0.30 + \frac{0.05}{0.95}\right)(180 - 55)$	73,500
	Useful drying $6{,}579 \times 1{,}127$	7,400,000
	Stack losses $6{,}579\ (475 - 212)\ 0.47 + 4{,}570\ (475 - 50)0.24 + 567 \times 17.8\ (475 - 50)\ 0.25$	2,350,000
	Total heat balance	11,032,500
(*t*)	Evaporated heat, in Btu per lb of H_2O evaporated $\frac{11{,}050{,}000}{6{,}579}$	1,680
(*u*)	Efficiency (%) $\frac{1{,}127}{1{,}680}$	67

Fuel.—The heat for drying sludge preparatory to incineration or use for fertilizer may be derived from burning the sludge itself or from some commercial form of fuel. In many incineration operations, the heat content of the sludge may prove sufficient to supply the needs for drying and maintaining a furnace temperature sufficiently high to assure complete deodorization. Under these conditions, auxiliary fuel is needed only during starting periods. Digester gas, if available, is an excellent fuel for this purpose. Where the supply of digester gas is intermittent and variable, it is advisable to supplement it with natural gas or fuel oil. City gas, if available, is also satisfactory, but its cost is usually prohibitive.

The quantity of fuel required for starting depends largely upon the size and type of furnace and the mass of refractory to be heated to the working

temperature. Some furnaces can be safely brought to working temperature in less than an hour, whereas some types require 24 hr to 72 hr, depending upon their size. Furnaces of the latter type also require the burning of fuel during the cooling down period, which varies between 24 hr and 48 hr.

For drying and incinerating sludge in an installation of moderate size, where supplementary fuel is needed frequently in regular operation, such fuels in order of their desirability, without reference to cost, are digester gas, natural gas, and fuel oil, or combinations thereof. In the larger installations, powdered coal is very satisfactory and economical. Where combinations of gaseous and liquid fuels are utilized, automatic means for change-over usually should be provided for protection against failure of either source of fuel supply.

Where sludge is processed for fertilizer and no sludge is burned, the entire heat requirement must be derived from some form of commercial fuel, or digester gas, if available. For the small or moderate-sized drying plant, as a practical consideration, it is imperative that gaseous or liquid fuels, or their combinations, be used. The first cost of such an installation is relatively low compared to an installation for burning coal. Usually the higher cost of gaseous and liquid fuels for this purpose over that of coal is offset by higher combustion efficiency, simplicity in operation, and flexible temperature control.

Where sludge is dried for fertilizer, the effect of the type of fuel on the final product is negligible. Therefore, with installations where large amounts of fuel are required, the relative economy of the fuel per unit of heat should determine the type. Coal burned in suspension or on a grate usually will be found more economical than other types of fuels in the larger installations, and when burned on a grate it may be the most economical for small units. Some of the advantages and disadvantages of various fuels are listed in Table 18.

Power Generation at Southwest Works

Whether the generation of power should be combined with drying and incineration of sludge is largely a question of economics. Such a combination ordinarily is not economical except for large installations like the Southwest Works at Chicago where conditions for the combination were very favorable. In the large flash type of drying system, with incineration furnaces, most of the elements of a steam generating plant, both as to equipment and personnel, are present.

At the Southwest Works, the annual costs of steam generation consist principally of fixed charges on the steam generating equipment plus fuel costs. A further advantage is inherent in the treatment plant design due to an abundant supply of plant effluent available for condensing purposes, at ample head to provide gravity flow through the condensers, before release to the outfall. Thus, the necessity for condenser circulating water pumps is eliminated at a considerable saving in the cost of power. The combined effect of these items results in a very low cost for power.

Steam for plant power purposes is generated in four units each of the water-tube, vertical, bent-tube type, combined with a waterwalled furnace in which dried sludge can be burned in suspension together with pulverized coal.

Each unit has a maximum steam generating capacity of 110,000 lb of steam per hour at a pressure of 425 lb per sq in. and a maximum temperature of 725° to 750° F. In addition to supplying heat for generating steam, each furnace

TABLE 18.—ADVANTAGES AND DISADVANTAGES OF VARIOUS FUELS

No.	Description	Coal Burned: In suspension	Coal Burned: On grates	Fuel oil	Natural gas	Digester gas
	(*a*) ADVANTAGES					
	Combustion:					
1	Combustion control is simple	√[a]	..	√[b]	√	√
2	High-flame temperatures easily controlled	√	..	..	√	√
3	High combustion efficiency	√	..	√	√	√
4	Heat value is high	..	..	√	√	√
	Fuel:					
5	Can be stored easily	..	..	√	..	..
6	Convenient to handle	..	..	√[c]	..	..
7	Available anywhere	..	..	√	..	..
8	Cost is low	..	√	..	..	..
9	Supply is reliable	..	√	..	..	..
10	Cost is stable	√	√[a]	..	..	..
	Plant and Equipment:					
11	Burning equipment simple	..	..	√[b]	√	√
12	Cost of burning equipment maintenance is low	..	..	√	√	√[d]
13	Furnace can be started easily	..	..	..	√	√
14	No fuel preparation plant is needed	..	..	..	√	√
15	First cost of installation is low	..	√	..	..	..
16	Cleanly	..	..	..	√	√
	(*b*) DISADVANTAGES					
	Fuel Supply:					
17	Not always reliable	..	..	..	√[e]	√
18	Not always available everywhere	..	..	..	√[e]	..
19	Diminishing, with resultant gradual increase in cost	..	..	..	√	..
20	Must be treated	√[f]	..	√[g]	..	..
21	Large storage space required	..	√	..	..	..
22	Excessive labor required for handling	..	√[h]	..	..	..
23	Costs may fluctuate	..	..	√[c]	..	..
24	High in sulfur compounds	..	..	..	..	√
25	Chemical contents variable	..	..	..	..	√
	Operation Problems:					
26	Combustion control is difficult	..	√	√[i]	..	..
27	Fire hazards high	..	..	√	..	..
28	Careful plant maintenance needed	√[j]	..	√[j]	..	..
29	Expert labor required	..	√[k]	..	..	..

[a] Relatively. [b] Compared with coal. [c] Particularly the lighter oils. [d] Exceptionally low. [e] Natural gas is not available in all sections of the United States, and where it is available, the supply is usually unreliable. [f] Coal must be pulverized with costly equipment, and preheated air is necessary. [g] Fuel oil must be strained and heavy fuel oils must be heated. [h] For coal and ash handling. [i] Compared with gas. [j] Burning equipment must receive careful attention and, with coal, fly ash wears the fans and ducts. [k] Requires competent firemen.

is capable of liberating sufficient heat for sludge drying to evaporate as much as 49,000 lb of water per hour at atmospheric pressure and of passing this quantity of water and other gases through the furnace for deodorizing. The total evaporation for each unit under maximum steam and average drying conditions is approximately 150,000 lb of water per hour, of which 110,000 lb is high-pressure steam and 40,000 lb is steam at atmospheric pressure.

Each furnace is equipped with six burners for pulverized coal and four for dried sludge, arranged for vertical firing through the top arch of the furnace.

A water-tube slag screen is provided in each furnace immediately above the hopper bottoms. The furnace volume of each unit, exclusive of back passes of the boiler and hopper bottoms, is approximately 8,000 cu ft. The total heat absorption surface of each boiler is approximately 13,875 sq ft, 3,185 sq ft of which is waterwall surface. The superheater, in addition, contains approximately 4,000 sq ft of surface.

The furnace gases which supply heat for the drying systems are taken off at the top of the furnaces and drawn through the vapor heaters by variable-speed induced draft fans. The gas then passes through dust precipitators for ash removal and thence to the stacks. Gases passing through the boiler and superheater are drawn through an air preheater by a variable-speed induced draft fan. They are exhausted from the preheater to an electrical precipitator where the ash carried in the gas stream is removed before release to the atmosphere by way of the stacks.

Sufficient heat is extracted from the gases in passing through the air preheater to provide primary and secondary combustion air at a temperature of approximately 400° F. Air for combustion is drawn directly from the boiler room by a forced draft fan discharging through the air preheater into a system of ducts where the proportion required for primary and secondary uses is automatically controlled.

The effectiveness of combining sludge drying with steam generation at this plant has been proved by nearly three years of operation. Aside from the production of cheap power, the arrangement reduces furnace maintenance costs due to the protection of the refractories by the waterwalls. Greater flexibility is afforded since furnace operation is not dependent on the drying load alone. More than half the fuel burned is for steam generation. Therefore, sudden changes in drying load do not upset furnace conditions as they would if the furnace were designed for drying only. With some combinations of filter-cake moistures and heat values of sludge, there will be more heat available in the sludge than is necessary for drying. Where drying only is done, this excess heat must be wasted to the stack; but if steam is generated, this heat can be utilized.

Conclusion

The intent of this paper has been to present, in an easily understandable manner, the major fundamentals relative to the drying and incineration of sewage sludge as has been developed to date in The Sanitary District of Chicago. Considerably more can be accomplished along these lines and it may be concluded that the art in its present stage will undergo many changes in developments over a period of time.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING

BY MERRILL BERNARD,[1] M. AM. SOC. C. E.

SYNOPSIS

This paper undertakes to report on certain progressive developments in applied hydrology, now a recognized field of engineering, and the science of meteorology, heretofore primarily associated with the prediction of weather and with the generalized influences of weather on the climatic régimes of a region.

Inadequacy in the quality, extent, and length of precipitation records has given incentive and justification to the development of more rational design techniques so that the flood to be accommodated by a structure may bear a direct relation to type and magnitude of the storm that will ultimately produce it.

Precipitation data for approximately 350 of the nearly 1,300 storms of record have been reviewed, the analysis beginning with the rainfall observers' notes. Of this number about 150 are presented in the paper, the tabulations including representative time-area-depth values.

The principal meteorological elements are defined. The distribution of moisture in the atmosphere and the dynamic action between air masses are discussed in sufficient detail to give basic understanding of the mechanism of storms.

Modern methods of water control and use have demanded a means for quantitative expression of the physical upper limits of storm rainfall. Efforts to meet this need are described, and although the studies which are the basis for this paper have been successful in achieving this end, they have also disclosed weaknesses in long-established techniques, inadequacies in quality and the extent of hydrometeorological data, and the need to readjust common concepts of flood-storm relationships.

It is the purpose of this paper to report on these relative successes and failures.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 1, 1943.**

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The Hydrologic Cycle Functionalized

The march of events that marks the progress of an elemental particle of water from the sea surface into the atmosphere, to the land, and back to the sea is known as the hydrologic cycle. In its passage two major influences are exerted—heat and the gravitational pull of the earth.

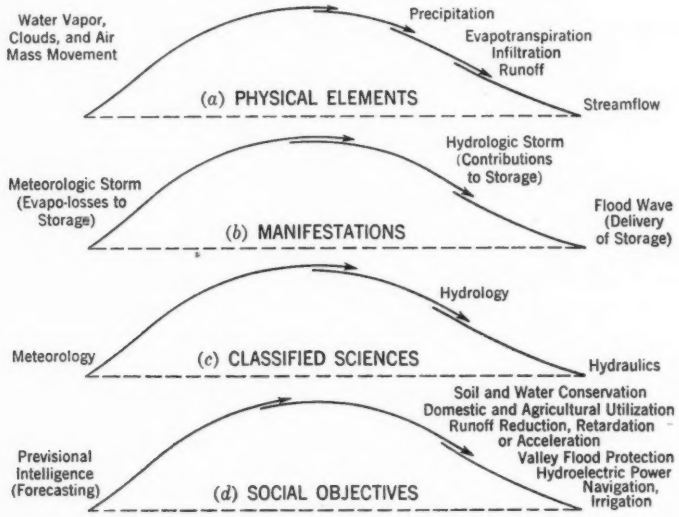


Fig. 1.—The Hydrologic Cycle Functionalized

Heat emanating from the sun is absorbed by the earth's surface and radiated back into the masses of free air moving over this surface. Heat then becomes the basic cause of turbulence in the air layers coming into contact with ground, vegetal, and water surfaces. Thus, through convectional displacement, repeated opportunity is provided for the absorption and removal of evaporated moisture.

The hydrologic cycle is not completely represented in the usually accepted sequence—from sea to land to sea. A particle of water evaporated from an ocean surface may be deposited very quickly at sea as part of a raindrop and so may never reach the land. Another particle may be taken into an air mass that moves to and over an extensive land area and out to sea again before meeting the influences necessary to convert it from vapor to liquid. Still another representative element in the train of moving water vapor may be deposited as rain or snow on the land, retaken into the atmosphere by evaporation, and transported back to the sea by air.

Much of the transitional water of the earth follows the more typical cycle that deposits the raindrop on the land surface, where it joins the runoff in overland flow, and enters a stream that ultimately flows into the sea. A modified cycle is that in which the raindrop enters the ground and reaches the stream later, through orifices below the flow line of its channel, whereas still

another drop may continue downward under the action of gravity to some deep ground-water reservoir from which it may ultimately reach a stream or percolate into the sea.

If all the moisture in the atmosphere were suddenly precipitated, there would be only enough to produce an average depth of about 1 in. of water over the total surface of the earth. What, then, accounts for the fact that rainfalls of more than 20 in. in 24 hr are experienced in many parts of the world? The answer is given by the meteorologist who observes and analyzes the factors known to bring about the convergence of vast air masses heavily burdened with moisture.

Fig. 1 has been prepared to show how the meteorologic, hydrologic, and hydraulic phases of the water cycle are related functionally as well as physically. Inescapable overlapping of interest is apparent and the need for coordinated effort becomes obvious.

It is believed that meteorology must play an increasingly important rôle in engineering planning. Although the science is not truly "exact," the meteorologist, nevertheless, deals with a medium for which well-established physical laws are known. His greatest limitation lies in the impossibility of sampling the vast extent of the atmosphere in a manner which even approaches statistical exactness.

Inadequacy of Flood Discharge Records

It would have been difficult before May, 1935, to convince the residents of Wakefield, Kans., that the Republican River could exceed a flood stage of 25 ft. It had risen from 12 to 21 ft in 1906, and to 25 ft in 1915, but for the

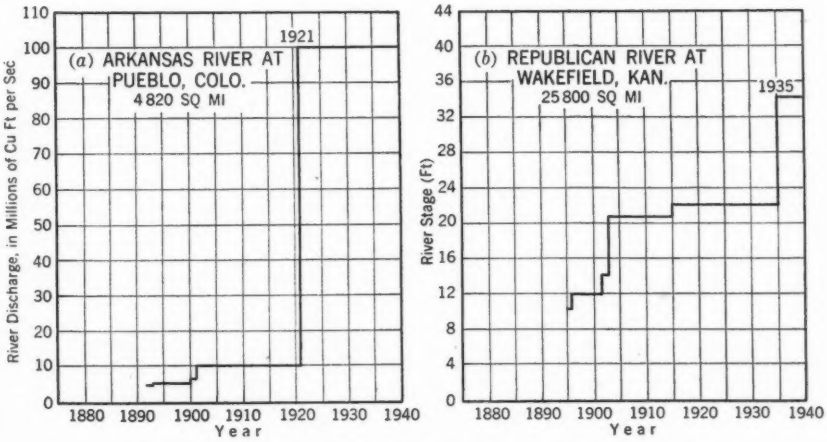


Fig. 2.—Maximum Flood of Record, to and Including Any Year

next 20 years the river failed to reach or exceed that stage. In May, 1935, the Republican River Basin was visited by a storm which produced a flood that registered 34 ft on the Wakefield gage. The prevailing maximum-flood stage, as it progressed throughout the period of record at Wakefield, and a similar

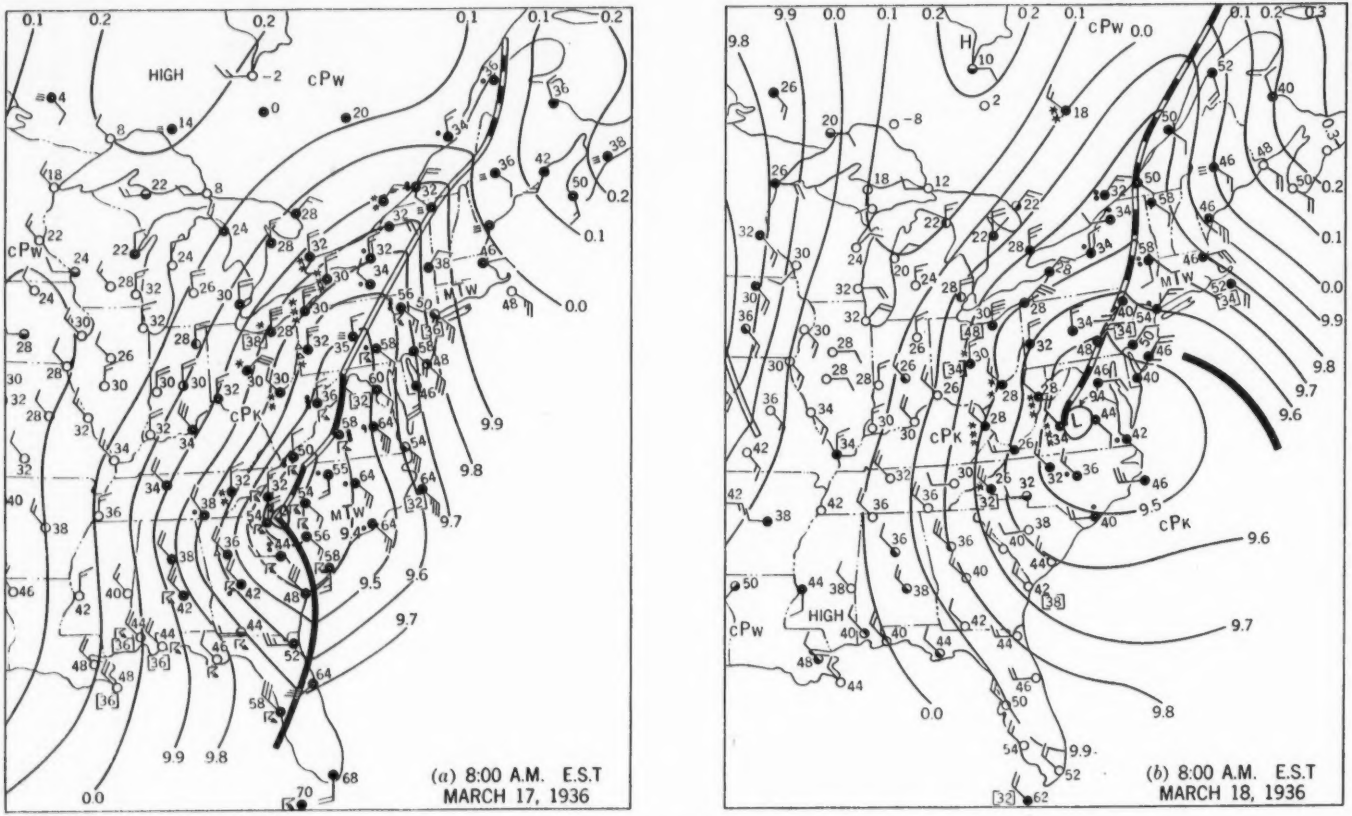


FIG. 3.—SYNOPTIC SURFACE WEATHER MAPS FOR A 24-HR PERIOD

FIG. 3.—SYNOPTIC SURFACE WEATHER MAPS FOR A 24-HR PERIOD

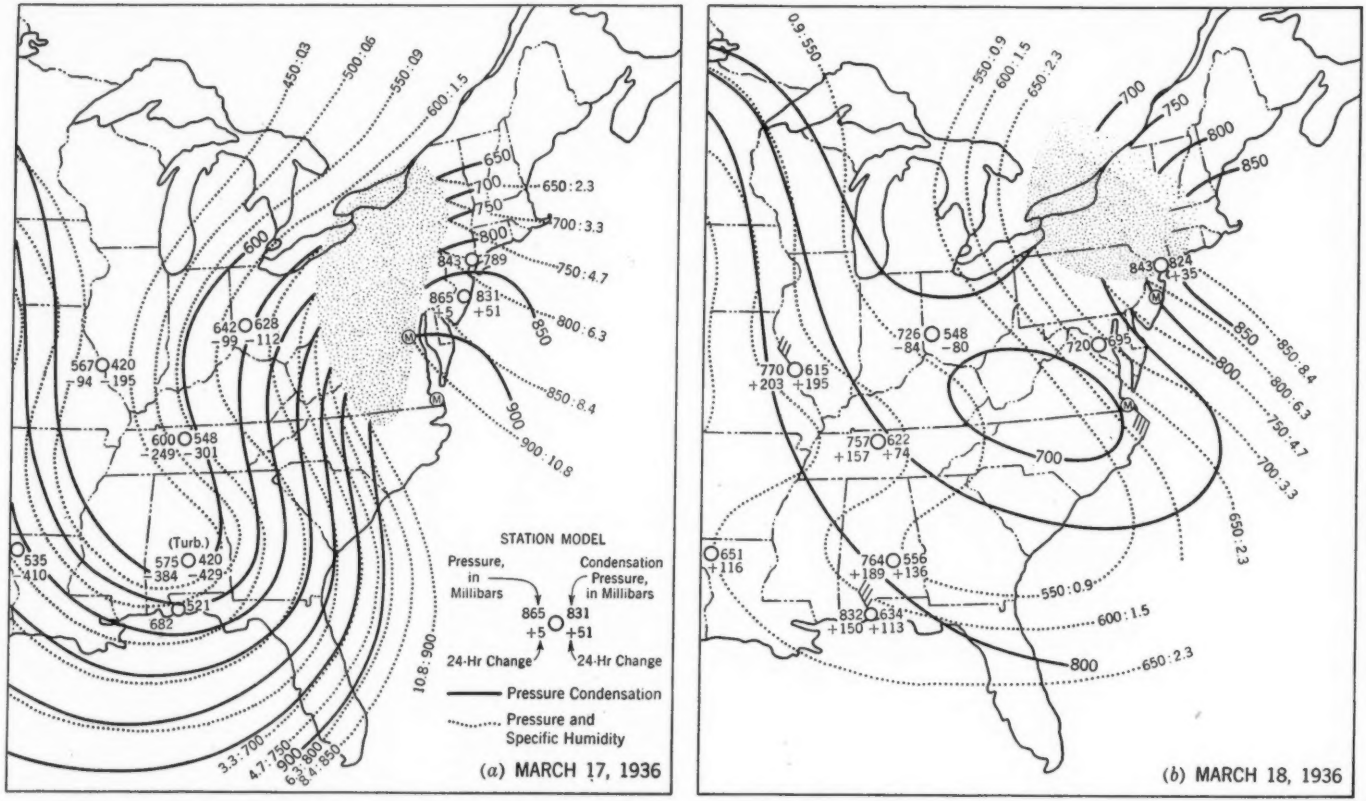


FIG. 4.—ISENTROPIC CHARTS FOR A 24-HR PERIOD; 8:00 A.M. (E.S.T.)

relation between increasing record length and maximum-flood stage at Pueblo, Colo., on the Arkansas River, is shown in Fig. 2.

The City of Wakefield will have established 100 years of record of stream flow in the year 1995. As this record progresses the prevailing maximum flood will approach the asymptotic limit represented by a flood which will result from the most critical combination of maximum-storm rainfall, snowmelt contribution, and minimum losses to storm runoff. The insecurity in estimating maximum-flood stage by extrapolating the curves in Fig. 2 accounts largely for the rationalized design methods developed by the Engineer Department of the Corps of Engineers, and others bearing a responsibility for flood-control design.

Inadequacy of Precipitation Records

A number of the larger cities in the United States have records covering approximately 100 years. These records are valuable in statistical studies relating rainfall intensity to duration, but are of limited value in problems involving the areal distribution of flood-producing rainfall over a basin. Only a few of the historical storms, that of August 5, 1843, in New Jersey and eastern Pennsylvania being an example, are sufficiently represented by data to permit the plotting of isohyetal maps. In the main, it has been necessary to make best use of the record period beginning about 1892, a conclusion also reached by the Miami Conservancy District in its classic work, "Storm Rainfall of Eastern United States" (1).[2]

Fig. 5.—Isohyetal Map for the 24-Hr Period Ending 8:00 A.M. (E.S.T.), March 18, 1936

This paper will not undertake to review the development of flood-control planning. It is sufficient to state that early attempts in utilizing recorded storm rainfall as a guide to the measure of future storms failed in answering the questions: "Could this storm, having occurred here as recorded, been greater in intensity, duration and

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

extent?" or, "Could this storm, having occurred here, be considered as representative of one which will ultimately center over a particular basin located outside its limits?" The meteorologist, whose field embraces the causes of precipitation, has been called upon to consider these questions.

The Storm as the Design Unit

Storm rainfall can be converted into flood flow in a rational and realistic manner. All of these rational methods require that rainfall be expressed in terms of average depth occurring within a fixed period of time, distributed over an area of known dimensions and pattern. Only rainfall recorded concurrently can become the basis for such analysis, a requirement seldom fulfilled by the older records which often fail to disclose the actual period during which the rain fell. It can be seen that if stations "A" and "B" are sufficiently close they will produce practically identical records of rainfall. As they are separated by increasingly greater distances the record at "B" becomes less and less dependent upon that at "A" until the records are no longer related and, therefore, cannot be combined even when statistically identical (4).

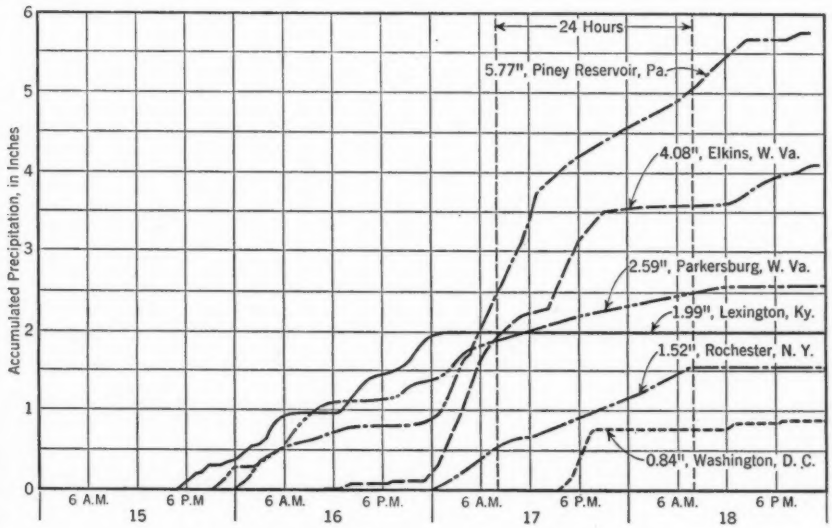


Fig. 6.—Mass Curves of Rainfall for March, 1936

Thus far the writer has treated only the "hydrologic storm," shown graphically as an isohyetal map and usually limited in total extent to areas less than 200,000 sq miles. The meteorologist, quite naturally, is interested in the hydrologic storm as it expresses the results of his analysis. However, he has been more directly interested in what will be referred to in this paper as the "meteorologic storm," a three-dimensional phenomenon of considerably greater extent than the hydrologic, or "rain" storm. His medium is the earth's atmospheric envelope—the field of action being the greater part of the northern

TABLE 1.—AREA-DURATION-DEPTH

Region	(a) NORTH

Locality	Pa., N. J., N. Y., and Conn.				Northeastern States				Eastern Pa. and N. J.			
Center	Smith's Corner, Pa.				Jewell, Md.				Camden, N. J.			
Date	September 18 to 20, 1894				July 26 to 29, 1897				August 3 to 5, 1898			
Area (sq miles)	100	1,000	10,000	40,190	100	500	2,500	26,241	100	500	5,000	30,600
Average Depth (In.):												
12 hours	4.9	4.0	2.6	1.6	12.9	8.9	4.1	1.9	4.9	4.5	3.2	2.4
24 hours	6.9	5.5	3.5	2.2	13.0	9.0	4.4	2.2	5.0	4.6	3.4	2.6
36 hours	7.7	6.3	4.4	2.9	13.8	9.6	4.6	2.5	6.2	6.1	4.6	3.6
48 hours	8.1	6.8	4.9	3.4	....	....	....	....	6.2	6.1	5.0	4.0
72 hours	....	....	....	....	....	....	....	....	....	....	....	....
Total Storm:												
Area	40,190 sq miles				26,241 sq miles				30,600 sq miles			
Duration	48 hours				96 hours				54 hours			
Depth	3.4 in. (average)				3.3 in. (average)				4.0 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(a) NORTH ATLANTIC

Locality	New England			New England			No. Atlantic States			New England		
Center	Somerset, Vt.			York, Pa.			Scituate and Westerly, R. I.			Provincetown, Mass.		
Date	November 2 to 7, 1927			August 20 to 24, 1933			September 16 to 17, 1932			September 14 to 18, 1933		
Area (sq miles)	2,500	25,000	100,000	100	500	2,000	500	2,500	5,000	500	2,500	10,000
Average Depth (In.):												
12 hours	5.8	2.6	1.4	11.3	9.4	7.8	....	....	....	2.4	2.1	2.0
24 hours	7.4	4.2	2.6	13.2	11.1	9.3	7.9	6.4	6.0	3.6	3.3	2.9
36 hours	8.4	5.1	3.3	13.2	11.1	9.3	....	....	....	5.6	4.9	3.9
48 hours	....	5.8	3.5	....	11.0	9.2	9.3	7.8	7.3	6.6	6.0	4.9
72 hours	....	....	....	....	....	....	10.8	8.7	8.1	9.7	8.8	7.3
Total Storm:												
Area	100,000 sq miles			5,000 sq miles			10,000 sq miles			10,000 sq miles		
Duration	48 hours			120 hours			48 hours			96 hours		
Depth	3.5 in. (average)			10.1 in. (average)			7.5 in. (average)			8.9 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(a) NORTH ATLANTIC REGION (*Continued*)						(b) SOUTH							
Locality	Pa., N. J., and Del.			New Jersey			Virginia and West Virginia				Central Maryland			
Center	New Jersey			Ewan, N. J.			Bloomery, W. Va.				Baltimore, Md.			
Date	August 19, 1939			September 1, 1940			September 28 to 30, 1896				July 12, 1903			
Area (sq miles)	100	1,000	10,000	100	500	2,500	500	2,500	10,000	54,467	100	500	2,500	4,280
Average Depth (In.):														
12 hours	15.8	11.3	4.6	18.4	13.1	7.4	5.5	5.1	4.4	2.6	...	...	....	....
24 hours	...	...	....	...	...	....	6.9	6.6	5.8	3.5	...	...	....	....
36 hours	...	...	....	...	...	....	...	...	....	3.7	...	...	....	....
48 hours	...	...	....	...	...	....	...	...	....	....	...	...	....	....
72 hours	...	...	....	...	...	....	...	...	....	....	...	...	....	....
Total Storm:														
Area	10,000 sq miles			2,500 sq miles			54,467 sq miles				4,280 sq miles			
Duration	18 hours			12 hours			120 hours				8 hours			
Depth	5.4 in. (average)			7.4 in. (average)			3.8 in. (average)				2.2 in. (average)			

Values for Selected Storms

Atlantic Region

Md., Del., Pa., N.J., and N.Y. Colora, Md. September 24 to 27, 1902				Pa., N. J., and N. Y. Dover, N. J. June 7 to 15, 1903			New Jersey Paterson, N. J. October 7 to 11, 1903				Atlantic States New Jersey July 18 to 23, 1919			
500	2,500	10,000	37,980	500	2,500	13,663	100	500	2,500	5,000	500	2,000	10,000	25,000
5.0	3.3	1.9	1.0	4.0	3.4	2.3	8.7	7.2	5.8	5.2	2.5	2.0	1.4	1.0
6.2	4.7	3.2	1.8	6.2	5.2	3.4	14.6	12.2	10.1	8.7	3.7	2.8	1.9	1.4
7.0	5.6	4.0	2.6	6.3	5.2	3.5	15.2	13.2	10.9	9.7	3.7	3.1	2.5	2.1
7.6	6.3	4.8	3.3	6.4	5.3	3.5	15.3	13.2	....	....	4.4	4.3	3.0	2.4
7.6	6.3	5.1	3.6	6.6	5.6	3.7	....	....	....	....	6.1	5.8	4.1	3.3
37,980 sq miles 72 hours 3.6 in. (average)				13,663 sq miles 216 hours 5.8 in. (average)			2,000 to 6,000 sq miles 48 hours				25,000 sq miles 120 hours 5.6 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)

Southern New York Hector, N. Y. July 6 to 10, 1935			New Hampshire Pinkham Notch, N. H. March 9 to 15, 1936			New England Pinkham Notch, N. H. March 17 to 19, 1936			New England Providence, R. I. December 19 to 21, 1936			Northeastern States Barre, Mass. September 17 to 21, 1938		
100	500	5,000	100	500	2,000	100	500	2,500	500	2,500	25,000	500	2,500	5,000
9.2	7.5	5.0	5.0	4.0	3.0	3.2	2.7	2.0	2.6	2.3	1.7	6.8	5.0	....
11.1	9.4	6.4	7.1	5.8	4.4	6.7	5.1	3.5	2.7	2.4	2.0	9.6	7.2	6.2
11.2	9.7	7.0	8.5	6.9	5.4	8.9	6.8	4.7	....	....	....	10.9	9.0	8.3
....	....	....	9.0	7.2	5.5	10.8	8.0	5.2	....	....	....	11.7	10.0	9.1
....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
5,000 sq miles 36 hours 7.0 in. (average)			2,000 sq miles 48 hours 5.5 in. (average)			2,500 sq miles 48 hours 5.2 in. (average)			50,000 sq miles 24 hours 1.8 in. (average)			40,000 sq miles 108 hours 8.3 in. (average)		

Values for Selected Storms (*Continued*)

Atlantic Region

S. E. No. Carolina New Bern, N. C. July 29 to 31, 1908			Eastern Pennsylvania Gordon, Pa. August 21 to 22, 1915				Florida Peninsula St. Petersburg, Fla. August 1 to 3, 1915			The Carolinas Altapass, N. C. July 13 to 17, 1916			
100	500	2,500	100	500	2,500	5,310	100	500	2,500	500	2,000	10,000	3,7000
3.7	3.6	3.4	4.7	4.5	3.8	3.1	13.6	10.6	6.8	10.5	9.1	5.5	2.2
6.0	5.7	5.1	....	....	....	....	14.3	11.8	9.1	16.6	13.5	8.6	3.8
7.2	6.8	6.2	....	....	....	....	14.5	12.5	10.0	18.8	15.5	9.9	5.6
7.9	7.6	7.1	....	....	....	....	15.2	13.2	10.5	19.5	16.3	10.6	7.0
....	....	....	....	....	....	....	....	....	....	20.1	16.8	11.0	7.8
15,000 sq miles 54 hours 5.9 in. (average)			5,310 sq miles 15 hours 3.2 in. (average)				10,000 sq miles 66 hours 8.3 in. (average)			37,000 sq miles 108 hours 8.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(b) SOUTH ATLANTIC										
Locality	S. E. No. Carolina			Md., Pa., and Va.				Florida				
Center	Hatteras, N. C.			Baltimore, Md.				New Smyrna, Fla.				
Date	September 14 to 16, 1917			October 9 to 10, 1922				October 4 to 12, 1924				
Area (sq miles)	500	2,500	10,000	100	500	2,500	12,350	100	500	2,500	5,000	8,579
Average Depth (In.):												
12 hours	5.0	4.4	3.2	4.4	3.6	2.5	1.7	18.3	14.8	8.6	5.6	3.2
24 hours	6.0	5.3	4.6	4.6	3.8	3.0	2.1	22.0	18.1	11.3	8.0	5.3
36 hours	7.0	6.4	5.6	....	....	....	....	23.4	19.4	12.7	9.5	7.1
48 hours	....	....	....	....	....	....	....	24.1	20.2	13.6	10.6	8.2
72 hours	....	....	....	....	....	....	....	24.7	21.1	14.6	11.7	9.3
Total Storm:												
Area	10,000 sq miles			12,350 sq miles				8,579 sq miles				
Duration	42 hours			27 hours				216 hours				
Depth	5.9 in. (average)			2.2 in. (average)				13.1 in. (average)				

TABLE 1.—AREA-DURATION-DEPTH

Region	(c) OHIO RIVER															
Locality	N. Ind. and Ohio, W. Pa., and Northern W. Va.				S. Ill., Ind., and Kentucky			Arkansas to Maine			Central West Virginia			
Center	Johnstown, Pa.				Fairfield, Ill.			Richmond, Ind.			Holcomb, W. Va.			
Date	June 14 to 18, 1912				June 14 to 18, 1912			March 23 to 27, 1913			March 10 to 15, 1918			
Area (sq miles)	500	2,500	25,000	49,330	500	2,500	50,000	500	2,000	25,000	500	2,500	10,000	17,495
Average Depth (In.):														
12 hours	3.1	2.5	1.4	1.0	3.0	2.5	0.8	4.7	4.1	2.8	2.9	2.6	2.1	1.8
24 hours	4.0	3.6	2.3	1.9	4.1	3.5	1.6	6.7	6.2	3.9	4.3	3.8	2.9	2.5
36 hours	4.2	3.8	2.6	2.2	4.2	3.6	1.9	7.6	7.0	4.7	4.7	4.2	3.2	2.7
48 hours	4.8	4.2	3.0	2.6	4.5	4.1	2.4	8.6	7.9	6.0	5.2	4.5	3.4	3.0
72 hours	...	....	....	2.8	4.8	4.3	2.7	9.8	9.0	7.1	5.8[a]	5.0[a]	3.8[a]	3.3[a]
Total Storm:											[a] Total storm duration, 66 hours			
Area	49,330 sq miles				62,775 sq miles			25,000 sq miles			17,495 sq miles			
Duration	120 hours				108 hours			96 hours			66 hours			
Depth	2.9 in. (average)				3.1 in. (average)			7.9 in. (average)			3.3 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(c) OHIO RIVER												
Locality	Ind., Ky., N. Y., Ohio, Pa., and W. Va.				Miss., Ala., Tenn., Ky., W.Va., and Ga.			Ky., Ill., Mo., Ind., Ohio, and Tenn.			Near East Central Ohio		
Center	Various centers				Rock Island, Tenn.			Earlington, Ky.			Near Keene, Ohio		
Date	September 1 to 4, 1922				March 21 to 24, 1929			June 21 to 22, 1935			August 6 to 7, 1935		
Area (sq miles)	500	2,500	10,000	22,972	500	2,500	25,000	500	2,500	25,000	500	2,500	19,297
Average Depth (In.):													
12 hours	4.8	3.4	2.2	1.4	8.1	7.0	4.1	7.6	6.0	2.3	8.7	6.4	2.9
24 hours	4.8	3.6	2.3	1.6	9.3	8.2	5.3	9.2	7.5	3.6	9.1	6.8	3.4
36 hours	4.8	3.7	2.9	2.4	9.6	8.6	5.6	....	....	....	....	....	3.4
48 hours	4.8	3.7	3.1	2.8	9.8	8.8	5.9	....	....	....	9.1	6.8	3.4
72 hours	4.9	3.9	3.3	2.9	....	....	....	....	....	....	....	....	....
Total Storm:													
Area	22,972 sq miles				122,070 sq miles			41,000 sq miles			19,297 sq miles		
Duration	84 hours				54 hours			42 hours			48 hours		
Depth	2.9 in. (average)				3.9 in. (average)			3.5 in. (average)			....		

Values for Selected Storms (*Continued*)

Region (*Continued*)

(*c*) Ohio River Region

Alabama and Georgia Glenville, Ga. September 23 to 28, 1929				Fla., Ga., N. C., and S. C. Vernon, Fla. September 29 to October 3, 1929			South Atlantic States Keysville, Va., and Swansboro, N. C. August 10 to 17, 1940			Arkansas to Ohio Golconda, Ill. October 5 to 7, 1910			
500	2,500	50,000	100,000	500	5,000	50,000	500	2,500	23,536	500	2,500	25,000	75,000
11.6	9.6	1.9	1.0	7.4	5.7	3.1	7.8	6.2	2.7	5.7	4.4	3.4	2.6
13.9	11.6	3.8	2.4	9.7	8.0	5.3	10.4	9.0	4.6	8.2	6.8	6.0	4.8
15.8	13.6	5.2	3.1	13.0	10.2	7.2	10.6	9.7	6.1	10.1	8.5	6.7	5.5
17.3	14.9	6.4	3.9	....	10.5	7.9	14.7	12.5	7.4	12.7	11.0	8.5	6.2
18.5	16.2	7.0	4.4	....	....	....	....	....	....	14.5	13.2	10.4	7.8
100,000 sq miles 120 hours 5.4 in. (average)				70,000 sq miles 60 hours 7.7 in. (average)			170,000 sq miles 186 hours 6.4 in. (average)			75,000 sq miles 90 hours 7.8 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)

Louisiana to Illinois Perryville, Tenn. March 14 to 17, 1919				Texas to Ohio Rolla, Mo. October 26 to 30, 1919			Western Virginia and North Carolina Marion, N. C. October 29 to November 2, 1921			
500	2,500	25,000	100,000	500	2,500	50,000	500	2,500	10,000	24,274
....	....	....	....	....	....	....	4.4	3.1	1.9	1.1
8.1	6.0	....	....	5.8	4.9	....	5.0	3.6	2.7	2.0
....	....	....	....	....	....	....	5.1	4.0	3.4	2.5
10.0	8.9	6.4	4.5	9.5	8.9	5.2	6.1	4.8	4.3	3.5
10.6	9.6	6.7	4.8	10.4	9.6	6.0	6.4	5.1	4.7	3.9
100,000 sq miles 96 hours 4.9 in. (average)				100,000 sq miles 120 hours 5.2 in. (average)			24,274 sq miles 96 hours 4.0 in. (average)			

Values for Selected Storms (*Continued*)

Region (*Continued*)

Ohio Newcomerstown, Ohio August 6 to 7, 1935			Texas to Pennsylvania Taylorsville, Ky. January 20 to 25, 1937				Arkansas to Ohio Fords Ferry, Ky. March 28 to 31, 1938			Kentucky Simpson Post Office, Ky. July 4 to 5, 1939		
100	500	2,000	500	2,500	25,000	100,000	500	2,500	10,000	100	500	5,000
10.4	8.7	6.4	3.6	3.2	2.4	2.0	4.4	3.4	2.4	14.4	10.5	5.2
....	....	....	5.9	5.3	3.9	3.3	6.7	5.4	3.6	....	....	....
10.9	9.1	6.7	8.0	7.1	5.6	4.8	6.7	....	....	....	....	....
....	....	....	9.2	8.6	6.9	5.6	6.7	5.6	4.2	....	....	....
....	....	....	11.4	10.5	8.7	7.3	8.0	7.2	5.9	....	....	....
2,000 sq miles 36 hours 6.7 in. (average)			100,000 sq miles 144 hours 9.7 in. (average)				25,000 sq miles 72 hours 4.5 in. (average)			5,000 sq miles 12 hours 5.2 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(d) GREAT LAKES											
Locality	N. Central Wisconsin			Northwestern Wisconsin				Wisconsin and Michigan			
Center	Butternut, Wis.			Medford, Wis., and Hart, Mich.				Merrill, Wis.			
Date	July 25 to 27, 1897			June 3 to 8, 1905				July 20 to 24, 1912			
Area (sq miles)	500	2,500	10,000	500	2,500	25,000	100,000	500	2,500	25,000	100,000
Average Depth (In.):											
12 hours	5.8	5.1	3.7	7.0	5.3	2.8	1.3	6.2	5.1	2.2	0.6
24 hours	9.0	7.8	5.3	7.6	6.2	3.9	1.8	10.2	7.7	3.7	1.2
36 hours	....	....	....	8.6	7.2	4.8	2.3	....	7.9	4.0	1.4
48 hours	....	....	....	9.3	7.8	5.0	2.5	....	7.9	4.0	1.4
72 hours	....	....	....	9.9	8.4	5.8	3.1	....	7.9	4.0	1.4
Total Storm:											
Area	15,000 sq miles			100,000 sq miles				100,000 sq miles			
Duration	66 hours			120 hours				120 hours			
Depth	4.7 in. (average)			3.4 in. (average)				3.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(d) GREAT LAKES REGION										
Locality	Central New York				New York and New England				New York		
Center	Syracuse, N. Y.				Severance, N. Y.				Philadelphia, N. Y.		
Date	June 9 to 12, 1922				September 11 to 16, 1925				October 17 to 20, 1930		
Area (sq miles)	500	2,500	10,000	20,371	500	2,500	10,000	13,825	100	500	1,000
Average Depth (In.):											
12 hours	4.2	3.5	2.1	1.7	2.1	1.9	1.6	1.4	4.2	3.1	2.4
24 hours	4.3	3.8	2.7	2.3	2.6	2.3	1.8	1.6	5.4	4.3	3.3
36 hours	4.9	4.0	2.8	2.5	4.2	3.4	2.4	2.0	5.7	4.5	3.6
48 hours	5.0	4.2	3.1	2.7	4.2	3.5	2.7	2.5	5.7	4.7	3.9
72 hours	5.2	4.4	3.4	3.0	....	....	....	2.7	....	....	....
Total Storm:											
Area	20,371 sq miles				13,825 sq miles				2,560 sq miles		
Duration	84 hours				126 hours				48 hours		
Depth	3.0 in. (average)				4.1 in. (average)				2.6 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(e) UPPER MISSISSIPPI								
Locality	Minnesota and Wisconsin				N. W. Minnesota and Eastern North Dakota				E. Kansas and S. E. Nebraska
Center	Pine River Dam, Minn.				New Folden, Minn.				Salina, Kans.
Date	June 2 to 6, 1898				July 1 to 6, 1901				May 28 to 31, 1903
Area (sq miles)	100	1,000	5,000	28,877	500	2,500	25,000	51,170	2,000
Average Depth (In.):									
12 hours	5.4	4.7	3.2	1.4	5.4	4.3	1.5	1.2	....
24 hours	6.7	5.7	3.7	2.0	6.7	5.0	2.2	1.7	5.4
36 hours	7.3	6.3	4.4	2.3	7.0	5.2	2.4	2.0	....
48 hours	8.0	6.9	5.1	2.9	7.0	5.2	2.6	2.1	....
72 hours	8.7	7.5	5.5	3.2	8.9	6.4	3.6	3.1	....
Total Storm:									
Area	28,877 sq miles				51,170 sq miles				2,000 sq miles
Duration	102 hours				108 hours				24 hours
Depth	3.6 in. (average)				3.1 in. (average)				5.4 in. (average)

VALUES FOR SELECTED STORMS (*Continued*)

REGION

Near Central New York Greene, N. Y. June 30 to July 5, 1915				New York and New England Mount McGregor, N. Y. July 7 to 9, 1915				Near Western New York York, N. Y. May 14 to 20, 1916			
100	500	2,500	18,021	500	2,500	10,000	17,178	500	2,500	10,000	21,351
2.6	2.3	2.0	1.1	3.0	2.8	2.2	1.8	3.5	2.7	1.9	1.5
3.6	3.5	3.1	1.9	3.7	3.5	3.1	2.8	4.1	3.3	2.5	2.1
....	....	....	2.2	....	....	....	....	4.4	3.8	2.9	2.4
....	....	....	2.2	....	....	....	....	4.8	4.1	3.1	2.6
4.5	4.3	3.8	2.7	....	....	....	....	4.9	4.2	3.5	2.9
18,021 sq miles 150 hours 3.4 in. (average)				17,178 sq miles 66 hours 2.8 in. (average)				21,351 sq miles 162 hours 3.2 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

(*Continued*)

(*e*) UPPER MISSISSIPPI VALLEY REGION

Near Eastern New York Elks Park, N. Y. October 4 to 7, 1932			Western New York Buffalo, N. Y. June 17 to 22, 1937			S. Minnesota and Wisconsin Minneapolis, Minn. July 24 to 28, 1892				N. W. Minn. and Eastern North Dakota Lamberton, Minn. July 18 to 22, 1897			
500	2,500	28,948	100	500	9,890	500	2,500	5,000	24,360	500	2,500	25,000	100,000
5.8	4.0	2.6	4.6	3.4	1.3	6.4	4.9	3.8	1.6	4.4	3.8	2.8	1.5
7.6	5.9	4.1	5.1	3.8	1.6	6.9	5.4	4.4	2.2	6.0	5.2	3.6	2.1
8.5	6.7	4.7	5.1	3.8	1.6	7.0	5.5	4.5	2.3	....	....	....	....
9.1	7.4	5.3	5.1	3.8	1.8	7.0	5.7	4.8	2.9	6.5	5.8	4.1	2.7
9.1	7.4	5.3	5.1	3.8	1.8	7.3	5.9	5.0	3.1	7.6	6.8	4.8	3.2
28,948 sq miles 98 hours 5.3 in. (average)			9,890 sq miles 120 hours 3.4 in. (average)			24,360 sq miles 108 hours 3.4 in. (average)				100,000 sq miles 102 hours 3.4 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

VALLEY REGION (*Continued*)

Minnesota, Wisconsin, and Michigan Ironwood, Mich. July 20 to 22, 1909			Northern Minnesota and Wisconsin Beaulieu, Minn. July 18 to 23, 1909				Southern Minnesota New Ulm, Minn. July 13 to 17, 1916				West Central Illinois Galva, Ill. August 18 to 20, 1924		
500	2,500	25,000	500	2,000	20,000	60,000	100	1,000	2,500	23,700	500	2,500	25,000
5.3	4.4	2.4	10.1	8.4	2.1	1.5	7.0	5.7	4.4	0.9	5.7	4.3	1.8
7.1	5.7	3.0	10.6	8.7	2.8	1.9	7.0	5.8	4.7	1.5	7.8	6.1	2.6
10.0	7.9	4.2	11.1	9.4	4.0	2.8	7.1	5.9	4.8	1.7	....	6.3	3.2
10.5	8.4	4.6	11.1	9.5	4.7	3.1	7.1	5.9	4.9	2.0	....	....	3.7
11.5	9.0	6.0	11.4	9.8	6.4	4.0	7.2	6.0	5.1	2.2	....	....	....
60,000 sq miles 108 hours 4.4 in. (average)			60,000 sq miles 96 hours 4.4 in. (average)				23,700 sq miles 96 hours 2.4 in. (average)				28,629 sq miles 54 hours 3.6 in. (average)		

TABLE 1.—Area-Duration-Depth

Region	(e) Upper Mississippi Valley Region (*Continued*)			(f) Missouri			
Locality	Eastern Missouri and Western Illinois			Iowa			
Center Date	Crystal City, Mo. June 9 to 11, 1938			Woodburn, Iowa August 26 to 28, 1903			
Area (sq miles)	500	2,000	10,000	500	5,000	20,000	60,000
Average Depth (In.):							
12 hours	6.8	5.4	3.7	9.3	6.4	4.0	2.1
24 hours	7.2	5.9	4.2	11.2	7.8	5.2	2.8
36 hours	....	....	....	12.2	8.7	5.9	3.4
48 hours	....	....	....	....	....	....	....
72 hours	....	....	....	12.6[b]	8.8[b]	6.2[b]	3.8[b]
Total Storm:				[b] Total storm duration, 54 hours			
Area	10,000 sq miles			60,000 sq miles			
Duration	24 hours			96 hours			
Depth	4.2. in (average)			4.3 in. (average)			

TABLE 1.—Area-Duration-Depth

Region	(g) Lower Mississippi							
Locality	Texas to New England					Southern States		
Center Date	Thomasville, Ala. June 1 to 5, 1928					Elba, Ala. March 12 to 15, 1929		
Area (sq miles)	100	500	2,500	25,000	100,000	500	2,500	50,000
Average Depth (In.):								
12 hours	9.0	8.1	7.0	4.5	2.1	13.9	9.9	2.7
24 hours	12.2	11.7	10.8	6.7	3.2	18.1	13.8	5.4
36 hours	13.9	13.4	12.4	7.8	3.8	20.5	16.8	7.2
48 hours	14.9	13.7	13.6	8.9	4.7	23.5	18.6	8.1
72 hours	15.0	14.7	13.9	10.0	5.5	25.5	21.0	....
Total Storm:								
Area	100,000 sq miles					135,000 sq miles		
Duration	108 hours					60 hours		
Depth	6.6 in. (average)					7.3 in. (average)		

TABLE 1.—Area-Duration-Depth

Region	(h) Southwestern											
Locality	Texas					Oklahoma and Texas			Texas and Oklahoma			
Center Date	Hearne, Tex. June 26 to July 2, 1899					Chattanooga, Okla. May 22 to 26, 1908			San Marcos, Tex. December 1 to 5, 1913			
Area (sq miles)	100	500	2,500	10,000	25,000	2,500	10,000	50,000	500	2,500	25,000	50,000
Average Depth (In.):												
12 hours	12.0	10.7	7.7	4.9	3.3	4.4	4.1	3.2	....	....	....	....
24 hours	22.3	18.0	12.8	8.7	5.5	6.4	5.8	4.2	8.5	6.5	4.0	....
36 hours	27.2	24.0	18.2	12.1	8.8	8.3	7.8	5.8	....	....	....	....
48 hours	28.4	25.1	20.8	14.6	10.2	....	....	....	11.2	9.2	6.3	4.8
72 hours	30.6	29.4	23.3	17.1	12.0	....	....	....	13.3	11.8	....	....
Total Storm:												
Area	40,000 sq miles					50,000 sq miles			50,000 sq miles			
Duration	72 hours					96 hours			96 hours			
Depth	9.5 in. (average)					6.7 in. (average)			7.4 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

RIVER REGION

Iowa, Illinois, and Indiana Bonaparte, Iowa June 9 to 10, 1905				Kansas, Missouri, Iowa, and Illinois Bethany, Mo. July 5 to 7, 1909		
500	2,500	10,000	25,000	2,500	25,000	50,000
10.9	8.8	5.5	4.0	....	....	....
....	....	....	....	5.7	....	....
....	....	....	....	....	....	....
....	....	....	....	6.4	4.8	3.9
....	....	....	....	9.0	6.4	5.4
25,000 sq miles 12 hours 4.0 in. (average)				60,000 sq miles 108 hours 4.4 in. (average)		

(*g*) LOWER MISSISSIPPI VALLEY REGION

Louisiana Alexandria, La. June 13 to 17, 1886			Southeastern Louisiana Near Saint Francisville, La: March 20 to 22, 1926			
250	1,000	5,000	500	2,500	10,000	28,210
17.7	15.8	9.9	6.1	4.7	3.2	2.2
22.5	20.1	12.6	7.1	5.8	4.3	3.2
26.5	23.7	14.8	7.1	5.8	4.3	3.2
28.6	25.6	16.0	7.6	6.2	4.7	3.4
29.0	26.0	16.2	....	....	....	....
6,900 sq miles 100 hours 14.7 in. (average)			28,210 sq miles 66 hours 4.2 in. (average)			

VALUES FOR SELECTED STORMS (*Continued*)

VALLEY REGION (*Continued*)

Arkansas, Louisiana, and Texas Logansport, La. July 22 to 27, 1933					Tex. to Ill. and Ky. Waldron, Ark. June 12 to 18, 1935			La., Miss., and Ark. Park, La. October 1 to 4, 1937			S. W. Louisiana Near Jennings, La. August 12 to 15, 1938		
100	500	2,000	20,000	100,000	500	2,500	50,000	500	2,500	25,000	500	2,500	25,000
11.4	10.5	8.5	4.1	0.8	6.3	5.2	1.8	8.9	7.2	2.7	10.7	8.3	2.8
16.9	15.7	13.0	6.5	1.6	7.2	6.0	2.6	11.9	10.0	4.4	13.1	10.2	3.9
18.7	17.4	15.5	8.6	2.6	7.7	6.5	3.1	13.2	11.3	5.1	....	....	....
19.4	18.5	16.9	10.9	3.4	7.8	6.9	3.7	14.4	12.0	5.3	13.7	10.8	4.7
20.6	19.7	18.4	12.4	5.6	8.4	7.6	4.9	....	....	....	....	....	....
100,000 sq miles 126 hours 7.3 in. (average)					102,450 sq miles 144 hours 5.0 in. (average)			25,000 sq miles 114 hours 5.4 in. (average)			34,210 sq miles 90 hours 4.3 in. (average)		

VALUES FOR SELECTED STORMS (*Continued*)

REGION

Texas and Arkansas Houston, Tex. August 16 to 21, 1915				Tex., La., Ark., and Mo. Houston, Tex. August 17 to 20, 1915			Arkansas and Tennessee Jessieville, Ark. April 20 to 21, 1927			Southwestern Texas Near Kerrville, Tex. June 30 to July 2, 1932			
500	5,000	50,000	100,000	1,000	10,000	50,000	500	2,500	5,000	Point	100	2,500	10,000
6.2	4.8	2.6	1.8	6.2	4.5	....	....	....	....	21.6	18.0	10.7	6.8
9.2	7.3	4.0	3.0	8.0	6.8	4.2	10.8	8.8	7.5	35.0	30.4	19.6	11.5
13.5	10.1	5.7	4.3	....	....	....	....	....	....	....	....	....	....
16.8	13.1	7.3	5.6	15.0	10.8	7.0	....	....	....	35.6	30.8	19.6	12.1
18.6	17.1	10.1	7.8	19.5	16.2	9.8	....	....	....	....	....	....	....
100,000 sq miles 120 hours 9.1 in. (average)				225,000 sq miles 96 hours 6.4 in. (average)			5,000 sq miles 24 hours 7.5 in. (average)			10,000 sq miles 48 hours 12.1 in. (average)			

TABLE 1.—AREA-DURATION-DEPTH

Region	(*h*) SOUTHWESTERN								
Locality	North Central Oklahoma			Oklahoma			Southern Texas		
Center	Enid, Okla.			Near Cheyenne, Okla.			D'Hanis, Tex.		
Date	August 5 to 17, 1932			April 2 to 4, 1934			May 31, 1935		
Area (sq miles)	500	2,500	9,500	100	500	5,000	100	500	2,500
Average Depth (In.):									
12 hours	8.1	5.9	3.4	16.7	12.2	5.1	16.6[c]	11.0[c]	6.1[c]
24 hours	9.5	7.0	4.5	....	....	....	....	....	....
36 hours	9.7	8.2	4.9	....	....	....	....	....	....
48 hours	10.1	8.6	5.2	....	....	....	....	....	....
72 hours	....	....	....	....	....	....	....	....	....
Total Storm:							[c] Total storm duration, 4 hours		
Area	9,500 sq miles			5,000 sq miles			5,000 sq miles		
Duration	48 hours			12 hours			4 hours		
Depth	5.2 in. (average)			5.1 in. (average)			4.6 in. (average)		

TABLE 1.—AREA-DURATION-DEPTH

Region	(*i*) NORTH PACIFIC REGION (*Continued*)						(*j*) SOUTHERN			
Locality	N. Idaho and N. W. Montana			Washington and Oregon			Southern California			
Center	Near Roland, Idaho			Cougar, Wash.			Opids Camp, Calif.			
Date	December 17 to 19, 1933			December 28 to 30, 1937			December 17 to 27, 1921			
Area (sq miles)	500	2,500	21,964	100	500	1,600	100	500	2,000	10,000
Average Depth (In.):										
12 hours	2.2	1.8	1.0	....	....	....	8.3	6.4	4.4	2.0
24 hours	2.6	2.1	1.3	....	....	....	14.0	10.6	7.4	3.4
36 hours	3.0	2.6	1.6	....	....	....	16.6	12.7	8.9	4.4
48 hours	3.7	3.2	1.9	10.5	8.8	7.2	19.3	15.9	11.6	5.9
72 hours	....	....	....	....	....	....	23.8	19.7	14.6	8.0
Total Storm:										
Area	21,964 sq miles			1,600 sq miles			10,000 sq miles			
Duration	60 hours			48 hours			234 hours			
Depth	2.1 in. (average)			7.2 in. (average)			15.0 in. (average)			

hemisphere. He has classified the interacting air masses on the basis of their heat and moisture properties, and has determined the manner in which one will oppose or yield to another. This generalized concept of cause and effect is illustrated in Figs. 3 to 6, in which the meteorologic storm represented by synoptic surface weather maps (Fig. 3) and isentropic charts (Fig. 4) is compared with the hydrologic storm expressed as an isohyetal map (Fig. 5) and mass curves of rainfall at selected points throughout the storm area (Fig. 6).

Beginning, then, with a general atmospheric disturbance of nation-wide proportions, the engineer-hydrologist moves progressively through the extensive area of general precipitation to the concentrations of rainfall of basin-wide extent. Here he assumes the responsibility for routing flood water over the ground as surface runoff and through the channel system as the flood wave.

VALUES FOR SELECTED STORMS (*Continued*)

REGION (*Continued*)

(i) NORTH PACIFIC REGION												
Southern Texas Rocksprings, Tex. June 12 to 15, 1935			Texas San Angelo and Carlsbad, Tex. September 13 to 17, 1936				Western Washington Wind River, Wash. December 16 to 19, 1917				Central Washington Elk Park, Wash. February 23 to 28, 1932		
500	2,500	10,000	100	500	2,000	5,000	500	2,500	10,000	15,675	100	1,000	10,000
8.1	6.7	4.4	16.5	13.3	8.8	6.8	3.2	2.6	1.9	1.7	4.5	3.3	2.3
8.6	7.4	5.4	22.6	17.5	10.7	7.7	4.8	4.0	3.1	2.7	8.8	6.7	3.8
8.8	7.6	5.5	22.8	18.5	12.1	9.0	6.3	5.3	4.1	3.5	12.0	9.2	4.6
9.8	8.7	6.5	22.8	18.6	12.2	9.3	8.1	7.1	5.6	4.9	13.1	10.4	5.2
....	....	....	....	....	....	....	10.3	8.7	6.7	5.9	16.2	13.2	6.4
10,000 sq miles 48 hours 6.5 in. (average)			25,000 sq miles 108 hours 10.1 in. (average)				15,675 sq miles 96 hours 6.5 in. (average)				10,000 sq miles 90 hours 7.5 in. (average)		

VALUES FOR SELECTED STORMS (*Continued*)

PACIFIC REGION

Southern California Hoegees' Camp, Calif. December 29, 1933, to January 1, 1934			Southern California Big Bear Lake, Calif. February 11 to 16, 1937			California, Nevada, and Oregon Sierra Slope December 9 to 12, 1937		
100	1,000	10,000	500	2,500	10,000	500	5,000	90,000
8.6	6.5	2.9	4.1	3.0	2.0	7.3	5.2	1.8
12.6	10.5	5.0	5.7	4.2	3.0	11.5	8.1	2.9
14.3	11.6	5.7	....	....	....	15.6	11.5	4.4
15.2	12.2	6.2	6.2	4.7	3.4	17.6	13.7	5.5
15.9	12.7	6.6	....	....	....	19.6	15.3	6.3
10,000 sq miles 72 hours 6.6 in. (average)			10,000 sq miles 48 hours 3.4 in. (average)			90,000 sq miles 72 hours 6.3 in. (average)		

HISTORICAL BACKGROUND OF STORM RECORDS

"Storm Rainfall of Eastern United States" (1) by the Miami Conservancy District, continues to be outstanding as an example of organized storm data. This excellent compendium, however, does not provide the details essential to the use of more recently developed techniques for storm analysis.

In 1937, when cooperative studies were begun by the Corps of Engineers, the U. S. Department of Agriculture, and by the Weather Bureau in the U. S. Department of Commerce, it was necessary to organize the precipitation data within individual storms. Since the greater number of records were based on observations made with the nonrecording rain gage, it became a major problem to identify the period within which the rain actually fell. The observers' notes in the manuscript records would occasionally indicate the time of be-

ginning and ending of rainfall, but the records of the comparatively few recording gages supplemented by a meteorological analysis of the synoptic weather maps for the period provided the means for making the most accurate distribution of cumulative rainfall depth throughout the duration of the storm.

Of the approximately 1,300 storms of record, about 350 have been analyzed in varying degree by the field offices of the Corps of Engineers and the Hydrometeorological Section of the Weather Bureau. Of this number 150 have been listed in Table 1. In addition to the generalized information giving the date and location of the storm, area-duration-depth values are included.

General Classification of the Principal Storm Types

The following classifications describe the basic actions which produce storm rainfall and, either singly or in combination, can be used to explain heavy rainfall in almost any part of the world. The primary requisites for intense rainfall are cyclonic convergent flow and a potential source of warm moist air. The addition of a lifting mechanism, such as a mountain barrier, or a wedge of cold air, enhances the opportunity for intense rainfall.

Type 1. Quasi-Stationary Front with Active Minor Waves.—A cold front that becomes quasi-stationary with sharp temperature and moisture contrasts between the air masses preceding and following the front, thus resulting in the development of active minor waves along the front.

Type 2. Rapidly Developing Waves Along a Cold Front.—Waves that develop rapidly into major cyclonic systems under favorable meteorological conditions.

Type 3. Major Occluded Cyclonic Systems.—Cyclones which, under favorable temperature and moisture conditions, develop into deep intense systems and cause cold polar air to sweep entirely around the system, thus cutting off the source of warm moist air and occluding the warm sector.

Type 4. Tropical Cyclones.—Storms of tropical origin which move inland, bringing with them unusual quantities of moisture to be precipitated as the storm slowly dissipates.

Type 5. Local or Frontal Thunderstorms.—

Local Thunderstorms.—Storms within an air mass resulting from convective overturning due to convergence, heating, or orographic lifting of moist, unstable air, but not associated with frontal activity.

Frontal Thunderstorms.—Storms produced by releasing the convective energy in moist, unstable, tropical air through frontal lifting.

Type 6. Moist Air Flow Up Mountain Slopes.—Storms resulting from steep pressure gradients which cause moist, unstable air to flow up mountain slopes.

A glossary of meteorological terms is presented as Appendix 2.

Regions of Meteorological Homogeneity

The mean annual isohyetal map for the United States shows the rainfall progressively decreasing from the seacoasts inland, indicating that its distance and direction from the source of moisture are important influences on the amount and duration of rainfall for a locality. Parts of the country having

marked departures in annual rainfall are usually regions of rugged topography—surface irregularities exercising an almost independent influence on rainfall. Differences in annual rainfall between regions of similar physiographic characteristics and moisture accessibility confirm the opinion that still other factors help to determine a rainfall régime.

Since there are no distinct boundaries of regions with records of consistent rainfall experience, it must be concluded that any combination of all factors causing rainfall is transitional, and regions of meteorological homogeneity can be outlined only after certain tolerances have been accepted. It is obvious that such regions, under any definition, must vary widely in extent. Great areas can be embraced within prescribed limits in the central third of the United States, whereas it may be impossible to assign boundaries of homogeneity to areas of appreciable extent in the quite rugged humid southeastern quarter of the country.

A region of meteorological homogeneity is defined as one in which all points will have the same, or reasonably similar, records of rainfall intensity and duration at the end of a significant period; or, it may be stated that a storm entering such a region can center over any point with equal probability.

Studies to date have not been sufficiently extensive to allow the exact delineation of all meteorologically homogeneous regions throughout the United States. A tentative subdivision, however, can be made in the 21 areas defined below:

Area	Region
1	Western side of the Pacific Coast Ranges
2	The lee side of the Coast Ranges
3	The Great Central Valley, California
4	The western side of the Cascades
5	The eastern half of Washington and Oregon
6	The plateau region of Utah, with emphasis on the local topographic features
7	Southern Arizona and California
8	The western slope of the Rockies
9	The eastern slope of the Rockies
10	Northeast Arizona and northwest New Mexico
11	Eastern New Mexico and extreme west Texas
12	Western Oklahoma, Kansas, and Nebraska
13	Dakotas and eastern Montana
14	Minnesota and northern Wisconsin
15	Central Mississippi Valley
16	Upper Ohio Valley
17	Western slope of the Appalachians
18	Northeastern States
19	Middle Atlantic Coast
20	Eastern Gulf States
21	Western Gulf States

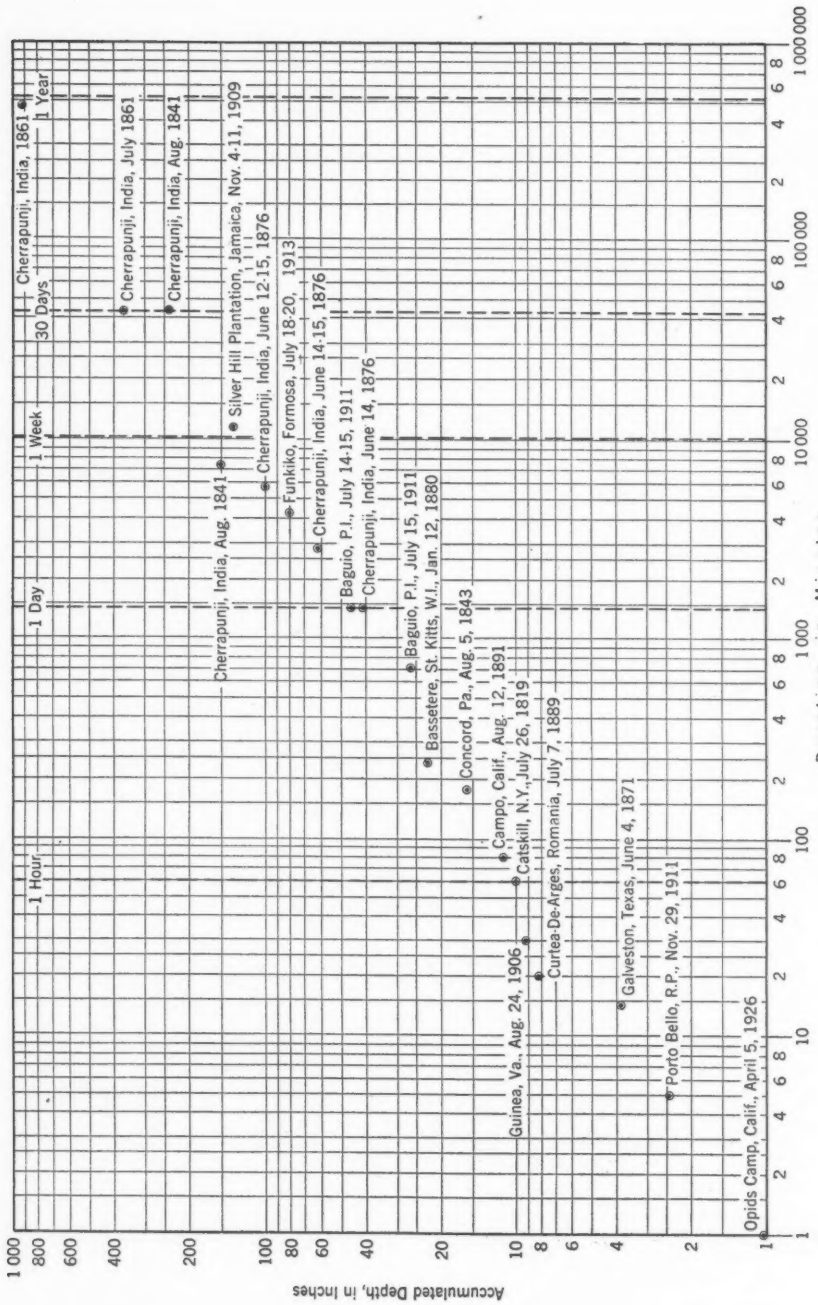


FIG. 7.—DURATION-DEPTH VALUES FOR POINT RAINFALL (WORLD'S RECORD)

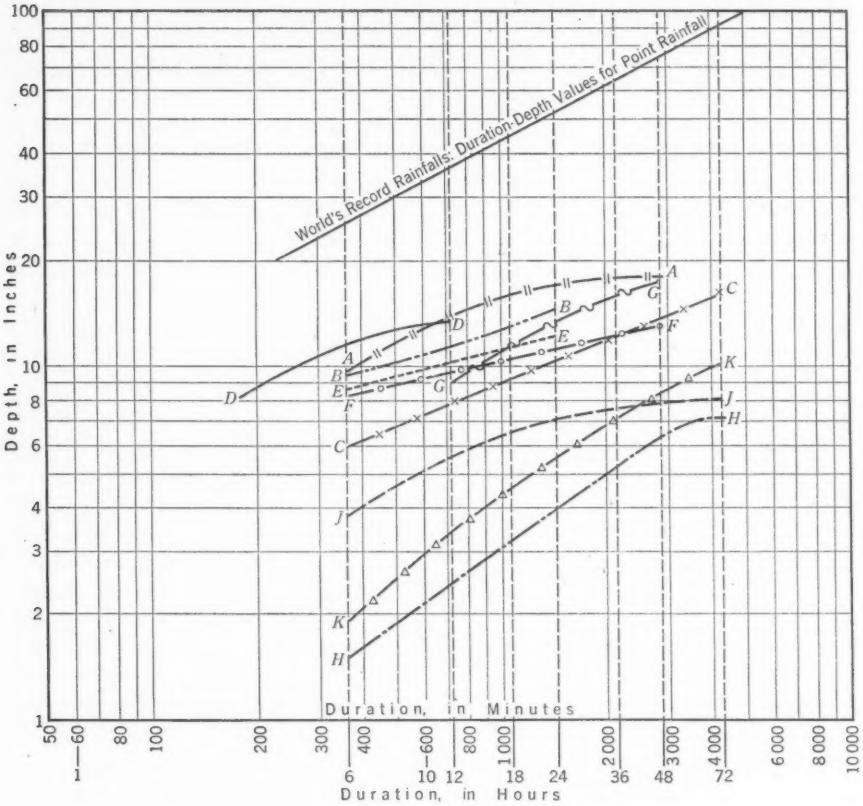
The Maximum Possible or Ultimate Flood

An immediate distinction to be made is that between the maximum possible flood and the maximum possible storm. Factors other than rainfall, such as melting snow, infiltration rates, and basin characteristics, enter into the determination of the maximum flood stage. The storm of September 11–13, 1878, produced more rain over smaller areas than did that of March 24–26, 1913. However, the former was the result of a decadent tropical hurricane which could not have occurred under conditions permitting a snow mantle over the upper Ohio Basin, whereas rainfall from a storm of the type and proportions of the latter could combine with snow melt under optimum cover and melting conditions to produce a flood of greater proportions. The maximum possible storm, then, is limited to that type which can produce the greatest amount of rainfall over a selected basin within a critical period fixed primarily by the concentration time for the basin and under conditions which are optimum for maximum snow melt and minimum infiltration and retardation.

Although rainfall amounts may be expected to increase as the record for a point or a small basin continues, it is a sound statistical inference that, within the average record period of about 50 years, every extensive region in the United States has experienced a storm which resulted from a critical combination of major factors and at least two of these have approached their physical upper limits. It is reasonable to assume also that at the end of an indeterminate period every basin in the region will have experienced the visitation of such a storm. The statistical evidence of these conclusions is supported by computations of maximum possible rainfall which are based on conservative theoretical assumptions.

Phenomenal precipitation records have been reported from various points over the world throughout the past 100 years. Their news interest has been greater than their value as records of flood-producing rains because no indication is given of the extent of the storm, of which they represent only a dimensionless point. Fig. 7 shows point rainfall depths accumulating throughout periods of varying durations. There is a well-defined upper limit. It should be noted that the lower range of the curve is well represented by values observed in the United States. Although a curve enveloping these points could be modified to represent average depth over specified area, its use in design would be extravagant in many cases, disregarding as it must the physiographic influences particular to the region in which the basin is located.

Limiting depth-duration curves for a number of basins in the United States are compared with that of world-wide point rainfall in Fig. 8. Differences in depth-range and curve-shape reflect differences in basin area and restraining regional influences which make it impossible for all basins throughout the country to have the same maximum rainfall experience. The primary objectives of the hydrometeorological studies have been to make it possible for the designer to consider economy by taking advantage of circumstances favoring a particular basin, while aiding him to gain security by disclosing the most critical storm-flood situation which his structure will ever be called upon to meet.



Curve	Basin	State	Area
A	Ompompanoosuc River above Union Village	Vermont	124
B	Mill Creek	Ohio	154
C	White River	Washington	400
D	Cherry Creek	Colorado	416
E	Youghiogheny River	Pennsylvania	430
F	St. Francis River above Wappapello	Missouri	1,310
G	Wolf Creek above Fort Supply	Oklahoma	1,660
H	Arkansas River above Caddoa	Colorado	19,000
J	Ohio River tributary above Pittsburgh	Pennsylvania	19,117
K	Red River above Denison	Texas-Oklahoma	38,400

FIG. 8.—LIMITING DURATION-DEPTH CURVES

PRINCIPLES OF STORM TRANSPOSITION

Storm transposition is not a new idea. It is a comparatively simple procedure to place an isohyetal map in critical position over that of a basin and convert the rainfall into a flood wave at a selected point. In so doing the designer has merely stated, "Had this storm occurred over the basin under the assumed conditions of runoff, it would have produced a flood of these pro-

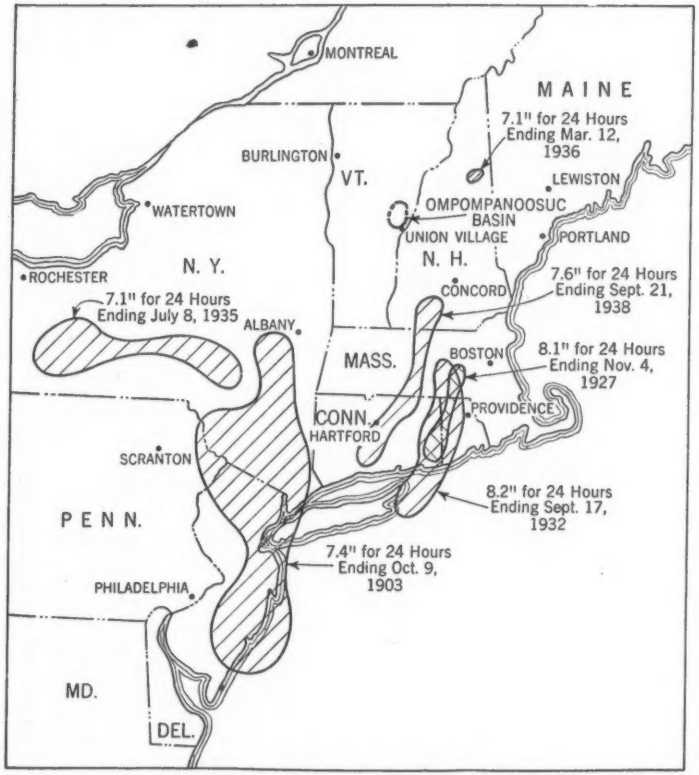


FIG. 9.—OMPOMPANOOSUC BASIN UPSTREAM FROM UNION VILLAGE, VT. Approximately Fifty Storms Were Reviewed or Analyzed, Seven of Which Became Control Storms

FIG. 10.—ST. FRANCIS RIVER UPSTREAM FROM WAPPAPELLO, MO. Note the Absence of Significant Storms in the Immediate Vicinity of the Basin; Fifty-Five Storms Were Investigated, of Which Eight Were Considered Critical

portions." He is not yet in position to know that the storm could have occurred within the season implied by the assumed conditions of runoff; nor that it could occur such a distance and direction from its actual location. Nor can he, if interested in maximum values, be sure that the storm could not have been much greater in magnitude.

To proceed beyond this point in reaching the objectives previously set forth, it is necessary to develop a philosophy which recognizes the limitations of basic data and scientific knowledge, strikes an intelligent balance between the empirical and the rational, resorts to theory as a guide in expressing qualified judgment, and in a common-sense fashion proposes to make the best use of whatever data are available—about 50 years of fairly dependable precipitation records, 40 years of acceptable synoptic weather maps, and a scanty 8 years of reasonably adequate upper-air data.

It is more the exception than the rule to find adequate record of hydrologic data on a basin designated for flood study. More often, it is necessary to anticipate the basin's future rainfall experience from an extensive study of the region in which the basin is located. It is known, to begin with, that the basin will ultimately (at the end of X years) produce a record of storm precipitation which will have in it one storm experience at or near the upper limit fixed by the characteristics of the region of which the basin is a part. If the basin approaches the region in size it will have this experience in a much shorter period than if it is small and storms entering the region have opportunity to center elsewhere.

As basins depending upon reservoir storage for flood control are usually much smaller than the region in which they are located, the procedure is usually to transpose to them those storms of record which, for a particular duration, have produced rainfall under conditions which are known to be supercritical. It would be possible, of course, to superpose all storms over the basin, plot average depth for the basin area for selected durations and obtain an enveloping curve, the increments of the curve becoming the synthetic design storm of typical pattern. The results of flood-storm analysis are given greater utility if area-duration-depth curves are prepared for the region which, under the assumptions of meteorological homogeneity, can be applied to any basin within it. In some instances it is necessary to transpose from one extreme to the other within a region, or between adjacent regions, in which case the transposed storm is adjusted to its physical upper limit for both the observed and transposed locations.

In Figs. 9 to 16 the assigned basin is shown in its relation to those storms contributing to or supporting maximum depth-duration values for the region. The depths indicated for each area are the average of actual rainfalls within a selected isohyet of major storms.

Theoretical Determination of Storm Rainfall

To determine theoretically the maximum possible precipitation over an area, it is necessary to develop an equation of flow. This equation expresses the obvious principle that the amount of precipitation is the difference between moisture inflow and moisture outflow.

FIG. 11.—OHIO RIVER UPSTREAM FROM PITTSBURGH, PA.

Two Hundred and Sixty-Five Storms Were Listed and Reviewed; of which Forty-Three Were Classified and Seven Were Adopted as Representative of Upper Limit Conditions within Their Respective Types

FIG. 12.—MILL CREEK BASIN, OHIO

One of the Early Studies Drawing Upon Storm Data Organized for Use in Other Reports

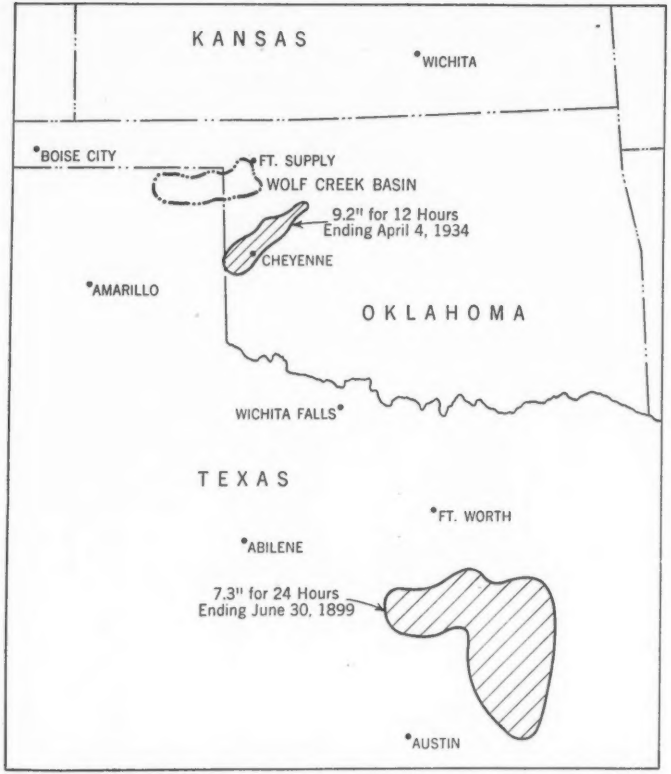


FIG. 13.—WOLF CREEK BASIN UPSTREAM FROM FORT SUPPLY, OKLA.
A Very Limited Number of Storms Had Been Analyzed at the Time This Report Was Prepared; Detailed Analyses Were Made of Only the Three or Four Most Significant

FIG. 14.—ARKANSAS RIVER BASIN ABOVE CADDOA, COLO.
One Hundred and Thirty-Three Storms Were Considered, of Which Six Contributed to the Maximum Possible Storm

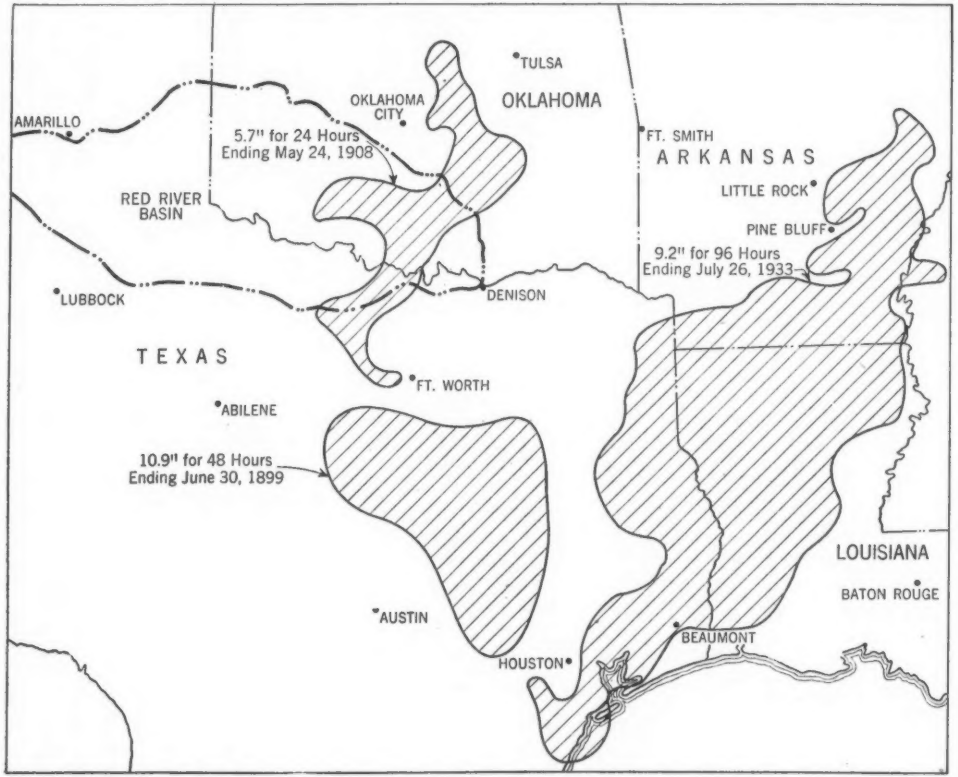


FIG. 15.—RED RIVER UPSTREAM FROM DENISON, TEX.
By Elimination, Nine Storms Were Found to be Significant, of Which the Three Shown Controlled the Limiting Depth-Duration Curve Developed in This Study

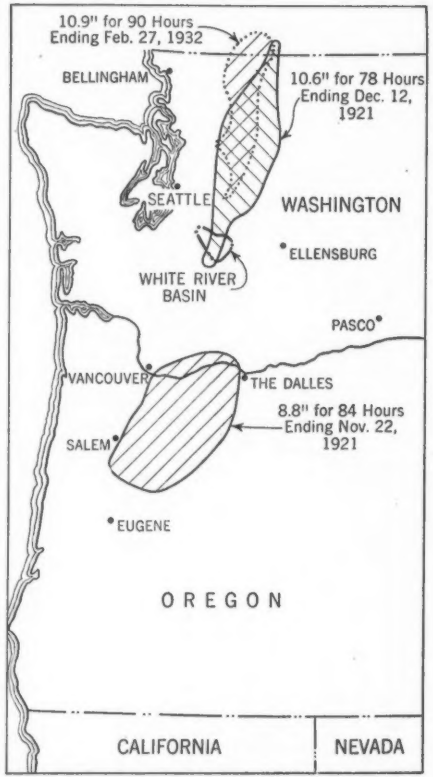


FIG. 16.—WHITE RIVER BASIN UPSTREAM FROM MUD MOUNTAIN DAM SITE, WASHINGTON
It was Necessary to Analyze Twenty-Six Storms in Order to Identify the Two Representing Maximum Storm Conditions for This Basin

It is evident that moisture inflow will be proportional to the horizontal component wind velocity. The wind velocity in the free air, of course, is a function of the density and the pressure gradient. From a study of thousands of upper-air soundings, it has been found that, at moderately high levels, there exists on the average a direct linear relationship between temperature and pressure. For example, at 5 km, at a pressure of 560 millibars the temperature is found to be 0° C, and at 495 millibars the temperature is − 50° C. This relationship, by the equation

$$\frac{p}{T} = R\rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

also fixes the density, ρ. (In Eq. 1: p = atmospheric pressure, in millibars; T = temperature, in degrees Centigrade; and R = the gas constant for dry air.) In the foregoing example, the density for 0° C and 560 millibars is 713 g per cu m; whereas that for − 50° C or 495 millibars is 773 g per cu m. Thus the extreme wind will depend, in general, on the extreme temperature gradients aloft. Without delving too deeply into the subject, it may be stated that with the present-day knowledge of the thermodynamics of the atmosphere the limits of extreme temperature aloft can be determined with reasonable accuracy. These limits are found to agree reasonably with observed data. There are also a vast number of pilot-balloon and anemometer observations that have been used in determining the maximum values.

To produce heavy rain two conditions involving moisture must be fulfilled: The air must be saturated; and the temperature-lapse rate must be such that rising air would be in unstable equilibrium.

These requirements definitely establish the upper limit of moisture content of a column of air as a function of the dew-point temperature at a fixed level. The moisture content of an atmospheric column, expressed in terms of water, is called precipitable water (W_P), which is the total amount of vapor in the column, and not the amount that can be removed. The problem now is to determine that which can be removed.

Nature has various ways of producing rainfall, the convective process being considered the most efficient. From a detailed study of the seasonal variation of all the phenomena associated with convective overturning, the functional relationships shown in Fig. 17 between the size of the convective cell and moisture charge have been derived. Surface dew point can then be taken as the measure of the amount of precipitable water in the atmosphere under the two conditions previously stated. The W_P-curve shows that with a dew point of 55° F at the surface there is 1 in. of precipitable water in the air column above, whereas a surface dew point of about 71° F accounts for 2 in. of water in the column.

The idealized model of a convective cell shows the existence of three layers—the inflow layer, the layer of maximum lift, and the outflow layer. The model to the left has a height of 18,000 ft conforming to a surface dew point of 50° F, whereas that to the right shows the cell size fixed by a dew point of 80° F. Intermediate values are represented by the curves.

Knowing the total precipitable water in the column and the total lift experienced, the amount removed as precipitation, and called effective precipitable water (W_E), can be computed. These values are shown by the curve in Fig. 17. For example, in the case of a dew point of 55° F the total water in the column is 1 in., but only 0.55 in. can be removed.

The flow equation, then, is:

$$R = \int K\, v\, W_E\, dt \qquad (2)$$

in which: K = a coefficient depending upon the basin characteristics; v = wind velocity in miles per hour; W_E = effective precipitable water in inches; and t = time in hours.

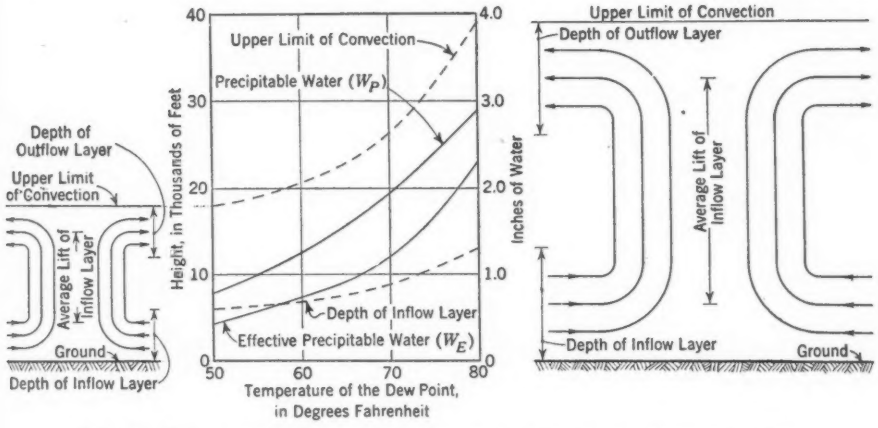


FIG. 17.—EFFECT OF MOISTURE CHARGE ON THE STRUCTURE OF A CONVECTIVE CELL

This means that the maximum precipitation will equal the product of the number of air columns moving into the basin and the maximum amount of moisture that can be squeezed out of each column by the most efficient natural process.

The minimum basin length for which the assumption of linear inflow is justifiable can be estimated from a comparison of the relative magnitudes of the vertical and horizontal components of the wind. If the width is approximately equal to the length, the minimum area is then about 1,000 sq miles for short durations, and about 10,000 sq miles for longer durations. A further restriction on potential rainfall is imposed by the maximum width of moist tongues which limits the moisture inflow column to a maximum of approximately 250 miles.

CONTRIBUTION OF SNOW MELT TO STORM RUNOFF

Precipitation in the form of snow acquires significance as a flood hazard when it accumulates over a basin beyond normal depth and water content. As a source of flood flow, snow can be considered a hydrologic element, but in

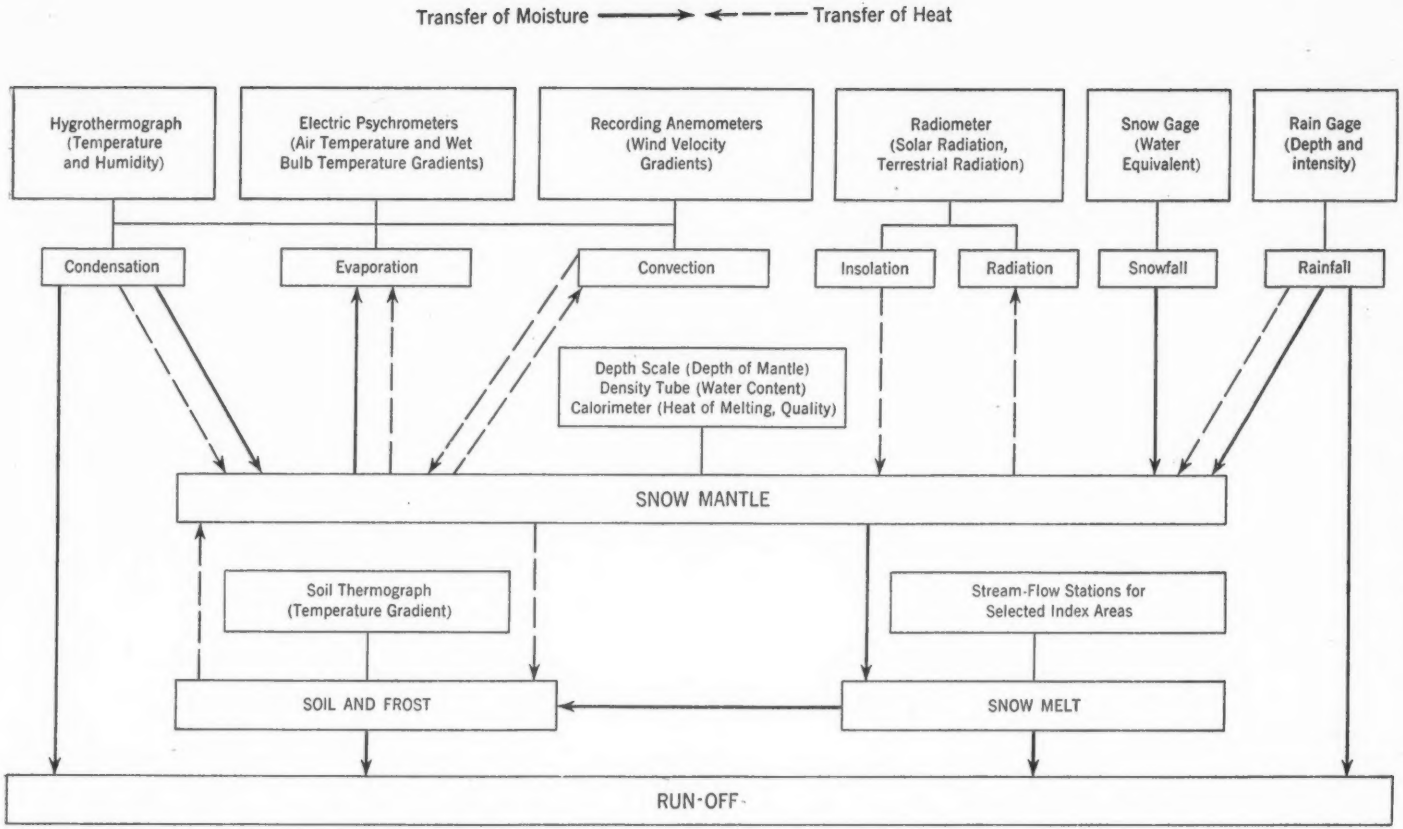


FIG. 18.—SNOW-HEAT-RUNOFF BALANCE; FUNCTIONAL RELATIONS AND INSTRUMENTATION

so far as the melting process is wholly involved with the factors, heat, moisture, and air movement, the problem more normally falls in the field of meteorology. The interrelationships between the atmosphere, the snow mantle, and runoff are shown in Fig. 18.

Engineers have made practical use of the fact that the temperature of the air is the most significant single factor in the melting process expressing this index in terms of degree-days above freezing. However, other sources of heat must be considered in any rational approach to snow-melt determination. These are—conduction from underlying soil, warm rain, radiation, condensation, and latent heat. Heat is transferred from the air to the snow surface through convection or turbulent exchange, a complex phenomenon which still defies exact analysis.

It is necessary, if the most critical snow-melt and runoff conditions are to exist:

1. That the snow is in a ripened state (that is, previous heat transfer has raised the temperature of the entire snow mass to the melting point; any further heating, therefore, produces melting);
2. That sufficient depth of frost exists in the underlying soil to check infiltration during the melting period; and
3. That rain occurs simultaneously with melting.

Under conditions (1) and (2) no temperature discontinuity exists at the soil-snow interface. This eliminates soil conduction as a factor in either melting or absorbing heat from the snow.

In ordinary circumstances, there is an appreciable heat exchange due to radiation. Snow absorbs a varying amount of solar energy or insolation depending on the character of its surface and angle of the sun. At the same time it radiates back to space as a "black body." Both the incoming and outgoing radiation are affected by the cloud cover which acts as a blanket to suppress radiational loss or gain. Continuous rain predicates an overcast sky and hence reduces the possible radiative heat transfer to a negligible amount.

Warm moist air transfers heat to the snow in two ways: By sensible heat exchange due to the temperature difference between the air and snow, and by the release of heat through condensation on the snow surface. This transfer of heat results in melting, or the conversion of the solid portion of the snow into liquid form at the rate of 1 g per 80 calories, the latent heat of fusion of ice. Since the heat of fusion of ice is only 80 calories and the heat of vaporization of water is 600 calories per g, the moisture condensed on the snow surface melts $7\frac{1}{2}$ times its own weight of snow.

In moving air the vertical flow of heat and moisture depends on the degree of turbulence of the air. According to the theory of atmospheric turbulence, the temperature, humidity, and strong wind movement measured near the ground are approximately logarithmic functions of height. Within the higher wind-velocity range, therefore, melting rates can be reasonably defined in terms of velocity, dry-bulb temperature, and dew point.

For maximum melting rates accompanying rainfall, the air is close to saturation and, accordingly, the dew point approaches the dry-bulb temperature. Hence, the wind velocity and either dew-point or dry-bulb temperature characterize the air layer next to the snow and can be used in the determination of melting rates throughout the course of a critical storm. As an example, a limiting value of 55° F has been found to be representative of air temperature over snow cover for the upper Ohio Basin, while the warm sector of the storm remains over the basin. This value is to be reduced as the larger basins are considered, due to the influence of the extensive snow cover on the temperature of the lower layer of the atmosphere.

It has been generally conceded, heretofore, that flood threat from snow melt is proportional to the average depth of snow or its water equivalent over a basin. However, it is believed that a more accurate criterion for maximum contribution from snow melt is the existence of only a sufficient amount of snow in storage to react to the melting influences throughout the entire passage of a storm over the basin—the storm to be characterized by extreme temperatures and humidities. The presence of more snow than can be melted under these critical conditions may serve as a deterrence to runoff in the early stages of the storm; or at least it may exercise a regulatory influence on runoff. As melting continues, successive areas of the basin become depleted of snow, causing a progressive decrease in the contribution of melt to runoff. The sequence of melting takes a characteristic pattern beginning with a relatively sharp rise as temperature increases from freezing to some maximum value which is followed by a gradual decrease as snow disappears over the basin. Thus it is evident that the storm and not the depth and distribution of snow determines the amount of snow-melt contribution to the maximum possible flood.

Procedure in Developing the Maximum Possible Storm for an Individual Basin

Four years of intensive study have disclosed no method for determining maximum possible storm rainfall which will apply consistently to all basins. The Hydrometeorological Section of the Weather Bureau, in complying with its assignments, has had to begin with the unproductive end of the problem; that is, it has had to deal progressively with (1) an area sufficiently great to insure the inclusion of the region of meteorological homogeneity embracing the basin; (2) the major drainage area containing the assigned watershed in which orographic influences are to be evaluated; and (3) the assigned tributary basins themselves, where the problem reaches its most difficult phase in that both area and time are reduced to units smaller than those inherent in the available storm and rainfall data.

To present the quantitative results of storm transposition in suitable form for predicting the maximum possible flood, use is made of a procedure, familiar in engineering practice, involving the use of a duration-depth curve. Values of maximum rainfall for arbitrary periods of the storm sequence are plotted against corresponding values of duration which then define an enveloping duration-depth curve of the particular storm. Successive increments of rainfall for convenient units of time may be read off this curve and then rearranged, sub-

ject to possible meteorological restrictions, according to a pattern that would produce the maximum flood peak for a basin. This particular time distribution of rainfall is referred to as the critical arrangement of a storm.

The prevailing techniques are best illustrated from a recent study and report on the maximum possible precipitation over the Ohio River tributary basins above Pittsburgh, Pa. In this case, the storm of March 22–27, 1913, became the key storm which, after modification and adjustment made necessary by transposition, contributed the controlling points of the limiting depth-area-duration curves. The storm of March 16–19, 1936, which caused the greatest flood of record at Pittsburgh, and the storm of January 20–25, 1937, similar in type to that of March 19, 1913, were analyzed in their actual and transposed positions.

A typical storm analysis leading to the determination of the maximum possible precipitation over a particular river basin is presented in that of March, 1913. This storm began with the rapid northeastward movement of a low pressure area and a relatively slow eastward movement of the cold front which finally became stationary over the northern tributaries of the Ohio River.

No upper air data are available for March, 1913, but by comparison with storms of recent years it can be inferred reasonably that there must have been an intense cyclonic circulation of cold air aloft over the Central Plains and upper Mississippi Valley region, and a strong west-southwest to east-northeast flow of moist tropical air over the Ohio Valley. The sharp contrast of heat properties between these two air masses made available large quantities of potential energy which could be utilized in releasing excessive amounts of precipitation over a narrow band. The boundary between the two air masses was obviously unstable, thus tending to be broken intermittently by intense wave developments of small magnitude. These caused sharply alternating fluctuations of cold dry air and warm moist air to move along the frontal zone. The large-scale, up-glide motion of the warm air over the cold air tended to prolong the rainfall, but it was the sudden lifting and intensely localized convergence associated with the minor waves which caused intermittent occurrences of intense rainfall.

Such minor waves appear to move in a general eastward direction, but it is almost impossible to predict at what point they will reach the maximum of their intensity. Also, in the extreme case a number of these waves may successively concentrate their activity at approximately the same point, thus accumulating a small but intense center of rainfall. Fortunately, this concentration of rainfall on small areas usually occurs at the expense of rainfall over the surrounding areas, and for basins as large as 20,000 sq miles it is reasonable to assume that in a period of 24 hr or less the concentration observed in March, 1913, was near the maximum.

In considering the transposition of the March, 1913, storm to the Ohio tributaries above Pittsburgh, there were two questions to be answered: (1) Could the storm have been greater in its original position? (2) How much increase or decrease should be allowed in transposing it?

The following steps were taken to answer these questions:

(1) Analysis of the weather charts on the dates of heaviest rainfall showed that a dew point of 64° F was representative of the moisture charge of the warm sector in the storm area. This corresponds to $W_e = 0.88$ in.

(2) The analysis of maximum possible dew points from the Gulf of Mexico to Pittsburgh for various seasons revealed that in late March in the vicinity of the 1913 storm, a dew point of 69° F ($W_e = 1.13$) was possible. In other words, the March, 1913, storm rainfall could have been 28% greater or 128% of the observed average depths.

Maximum possible dew points at Pittsburgh indicate a dew point of 63° F ($W_e = 0.84$) for late March. Therefore, the maximum possible storm of type I near Cincinnati, Ohio, would have to be reduced in the ratio of 1.13 to 0.84 in transposing it to the Pittsburgh area. This represents a 26% reduction in rainfall amounts.

It is realized that no statistical approach can provide a valid adjustment factor for type I storms occurring near Cincinnati and transposed to the Pittsburgh area. However, the comparative data in Table 2 are in general agreement with the fact that the maximum possible storm values will be greater near Cincinnati than near Pittsburgh.

TABLE 2.—COMPARISON OF AVERAGE RAINFALL (IN INCHES) AT CINCINNATI, OHIO, AND PITTSBURGH, PA.

No.	Description	Cincinnati	Pittsburgh	Ratio, Col. (3) / Col. (2)
	(1)	(2)	(3)	(4)
1	Annual precipitation	38.41	35.81	0.92
2	March precipitation	3.82	3.10	0.81
	Rainfall, Type I Storms:			
3	Greatest 35 storms	1.44	1.05	0.73
4	Greatest 10 Pittsburgh storms	2.73	2.14	0.78
5	Greatest 35, 12-hr storms	0.42	0.31	0.74

The value of 26%, computed by the dew-point technique, was accepted and used in the following adjustment: Considering the observed depths in the March, 1913, storm equal to 100%, the maximum possible type I storm in the same locality would be 28% greater or equal to 128%. However, this maximum possible storm must be decreased 26% when transposing it over the Ohio River above Pittsburgh. The maximum possible transposed storm therefore becomes 95% of the observed.

The same result can be reached quite simply by adjusting the observed 1913 depths to the maximum possible dew point at Pittsburgh. The effective precipitable water for the respective areas was 0.88 in. in the actual storm and 0.84 in. (the maximum possible) at Pittsburgh. This is a 5% decrease. Therefore, in transposing the 1913 storm to the Ohio tributaries above Pittsburgh a decrease of 5% was applied to all depths for all durations, on the assumption that the storm could have been greater in its original locality, but could not have been so great if the storm had centered over the Ohio tributaries above Pittsburgh.

In regard to the orientation of isohyetal patterns resulting from type I storms, it must be emphasized that such storms occur with upper-air currents from the west-southwest or southwest, and rainfall must necessarily take place

along a band which is at right angles to the major axis of the Pittsburgh basin. Since this particular direction of flow is necessary to bring moisture, unaffected by topography, into the basin, there can be no justification for changing the orientation of the original storm pattern. For the foregoing reasons a

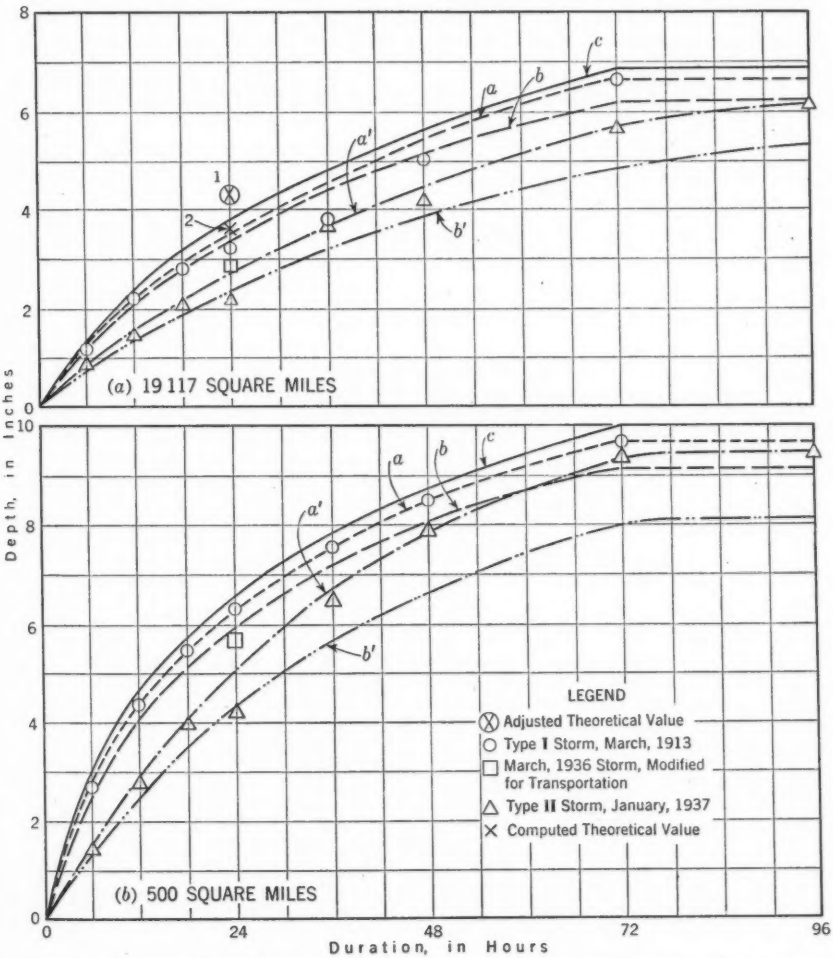


FIG. 19.—DURATION-DEPTH CURVES OF MAXIMUM POSSIBLE RAINFALL; STORM TYPES I AND II

mechanical transposition without re-orientation was considered most representative of the March, 1913, storm over the Ohio tributaries above Pittsburgh.

Two typical groups of duration-depth curves are shown in Fig. 19. Each block in the diagram contains a family of five curves applicable to the area of a particular subdivision of the basin. Two curves, one for each of the major storms classified as types I and II, represent duration curves of rainfall as it actually occurred over the upper Ohio Basin (curves a and b, Fig. 19). Two

other curves give corresponding values of rainfall obtained by transposing these storms to the most critical position over the basin (curves a′ and b′, Fig. 19). The enveloping curve is the duration-depth curve of maximum possible rainfall, representing as it does an upper limit fixed by actual values which have been increased with meteorological justification and modified according to the influences involved in transposing the storms to their assumed locations (curves c, Fig. 19).

Two additional 24-hr values have been plotted for the total basin (19,117 sq miles). One (Point 1, Fig. 19(*a*)) is the value computed from theoretical considerations involving moisture charge and velocity of air inflow. The other (Point 2, Fig. 19(*a*)) is the value which recognizes a possible error of 20% in the theoretical computation.

The maximum contribution to runoff from snow melt will result from a combination of maximum melting rates, maximum storage of free water in the snow and a uniformly deep snow cover over the basin. A series of snow-melt increments (see Fig. 20(*a*)) have been derived from principles previously de-

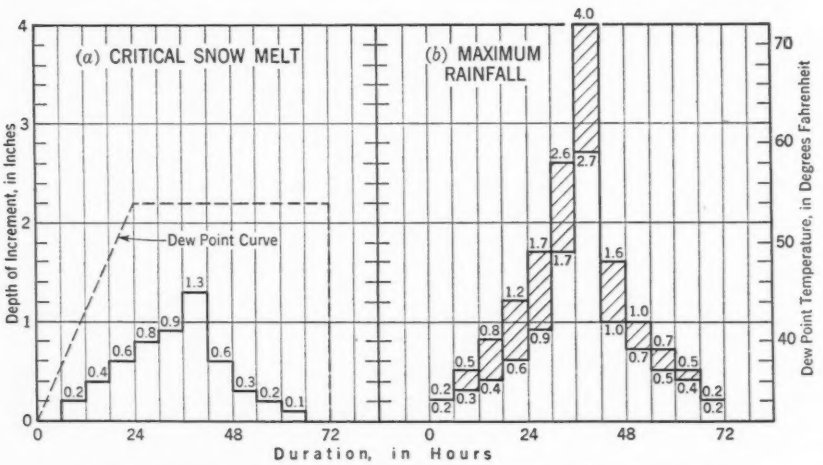


FIG. 20.—TYPICAL EXAMPLE OF PLOTTING DATA; MAXIMUM POSSIBLE RAINFALL (PLUS SNOW MELT); TYPE I STORM IN THE CONEMAUGH RIVER BASIN

scribed, assuming the distribution of snow cover which will respond most actively to the series of meteorological events characterizing the maximum possible storm.

The increments of maximum possible rainfall are now combined with those of snow melt to form the synthetic maximum possible storm shown, for the Conemaugh River Basin (as a typical example), in Fig. 20(*b*). The rainfall blocks have been arranged in an order which associates the highest rate of snow melting with the highest rate of rainfall, both being compatible with the meteorological features of storms of this type. Since the first 24 hr of the storm sequence is fixed by the meteorological situation and the accompanying snow melt, the rainfall-snow-melt increments of this period should not be inter-

changed among themselves or with those of the last 48 hr of the storm. Increments within the last 48 hr of the storm, however, may be interchanged as desired. The individual snow melt and rainfall increments must be considered inseparable, the necessity for which is readily seen from a consideration of the snow-melt computation procedure. These restrictions apply to all sizes of drainage basins.

The Bibliography in Appendix 1 will serve as a guide to selected reading for those whose interest in meteorology has carried them beyond the scope of this paper.

The writer should like to close by expressing his conviction that an engineer is not prepared to deal adequately with precipitation, the principal hydrologic factor, until he has gained a working knowledge of the processes which take from the sea, transport, and precipitate over the land the myriad of rain drops constituting a great storm. Thus equipped he will find his concepts broadened, his techniques made flexible, and his judgment strengthened.

Acknowledgments

The hydrometeorological studies described herein are conducted for, and in cooperation with, the Engineer Department, Corps of Engineers, U. S. Army. The writer wishes to acknowledge with thanks the valuable assistance of Albert K. Showalter, Samuel B. Solot, Phillip Light, Alexander L. Shands, and other members of the Hydrometeorological Section, Weather Bureau.

APPENDIX 1

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QE 500 .A512

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QE 500 .A512

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APPENDIX 2

Glossary of Meteorological Terms

Synoptic Chart.—A chart, such as the ordinary weather map, which shows the distribution of meteorological conditions over an area at a given moment.

Anticyclone.—An area of high barometric pressure and its attendant system of winds (clockwise in the northern hemisphere).

Cyclone.—An area of low barometric pressure with its attendant system of winds (counterclockwise in the northern hemisphere).

Air Mass.—An extensive portion of the earth's atmosphere approximating horizontal homogeneity.

Front.—The boundary between two different air masses.

Polar Front.—The surface of discontinuity separating an air mass of polar origin from one of tropical origin.

Cold Front.—Front at which relatively cold air displaces warmer air.

Warm Front.—Front at which relatively warm air displaces colder air.

Quasi-Stationary Front.—Front whose movement is such that there is no appreciable displacement of warm air by cold air or vice versa.

Cyclogenesis.—Process which creates or develops a new cyclone, or which produces an intensification of an old one.

Frontogenesis.—Process which creates a front or intensifies an existing front.

Frontolysis.—Process which tends to weaken or destroy an existing front.

Occlusion.—Process whereby the air in the warm sector of a cyclone is forced from the surface to higher levels.

Occluded Front.—The type of front resulting when a cold front overtakes a warm front.

Cold Front Type Occlusion.—Occlusion in which cold air in rear of the cold front is colder than and underruns the cold air in advance of the warm front.

Warm Front Type Occlusion.—Occlusion where cold air in rear of cold front is warmer than cold air in advance of warm front.

Isentropic Chart.—Synoptic chart plotted on a surface of constant potential temperature, and is so called since a surface of constant potential temperature, if unsaturated, is also a surface of constant entropy.

Potential Temperature.—Temperature which an air parcel would have if expanded or compressed dry adiabatically to a standard pressure of 1,000 millibars.

Equivalent Potential Temperature.—The potential temperature of an air parcel after all the latent heat of condensation of the contained water vapor is realized.

Dew-Point Temperature.—The temperature at which, under ordinary conditions, without change of pressure, condensation begins in a cooling mass of air.

Wet-Bulb Temperature.—Lowest temperature to which air may be cooled by evaporating water into the air at the same barometric pressure.

Specific Humidity.—Ratio of the weight of water vapor to that of air in a given sample, in grams per kilogram.

Lapse Rate.—Rate of decrease of temperature in the atmosphere with height.

Dry Adiabatic Lapse Rate.—Rate at which an ascending body of unsaturated air will cool due to adiabatic expansion, equal to approximately 1° C per 100 m.

Pseudo-Adiabatic Lapse Rate.—Lapse rate equal to the rate at which an ascending body of saturated air will cool during adiabatic expansion.

Its value is not the same under all conditions, but is never greater than the dry adiabatic rate.

Precipitable Water.—Total water vapor contained in an atmospheric column of unit cross-section area expressed in terms of a column of water of the same cross-section area.

Effective Precipitable Water.—The greatest amount of precipitable water which can be removed from an atmospheric column by convective action.

Tongue.—A projecting stream of air which may be dry or moist, revealed by the isentropic chart or atmospheric cross-section charts.

Trough.—An elongated area of low barometric pressure.

Stability.—Thermodynamic state in which vertical displacements are resisted.

Instability.—Thermodynamic state in which vertical displacements are favored.

Convective Process.—Process, mechanical or thermal, causing the upward or downward movement of a limited portion of the atmosphere. Cumulus type clouds are indicative of the occurrence of convective processes.

Level of Free Convection.—Level in the atmosphere at which, when reached, an air parcel is warmer than its surroundings and will continue to be displaced upward.

Lifting Condensation Level.—Level in the atmosphere at which, when reached, an air parcel will cool at the pseudo-adiabatic rate.

Conditional Instability.—Thermodynamic state which is unstable for saturated, but stable for dry or unsaturated, air.

Convective Instability.—Thermodynamic state of a layer which will become unstable after sufficient lift.

Up-Slope Winds or Up-Glide Winds.—Winds occurring when there is a net flow of air moving up a slope. When the slope is a front, the warm air moves up with respect to the colder air beneath the front.

Down-Slope or Down-Glide Winds.—Winds occurring when there is a net flow of air moving down a slope.

Orographic Rainfall.—Rainfall caused by the interference of rising land in the path of moisture-laden wind. A horizontal air current striking a mountain slope is deflected upward and the consequent dynamical cooling associated with the expansion of the air produces rain if the air contains sufficient aqueous vapor.

Convergence.—Condition that exists when distribution of winds within a given area is such that there is a net horizontal inflow of air into the area.

Gradient Wind.—The wind resulting from the balance (neglecting friction) between the force due to the pressure gradient, the apparent deflecting force of the earth's rotation, and the centrifugal force due to the curvature of path.

Geostrophic Wind.—The wind resulting from the balance of the force due to the pressure gradient and the apparent deflecting force of the earth's rotation, neglecting curvature of path and friction.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EFFECT OF VARIATION OF ELASTIC CHARACTERISTICS ON STATIC UNKNOWNS

Discussion

BY A. HRENNIKOFF, ASSOC. M. AM. SOC. C. E.

A. HRENNIKOFF,[9] ASSOC. M. AM. SOC. C. E.[9a]—In Fig. 8 Professor Finlay has plotted the graphs of the functions under the integral signs in the expressions for the variations of static unknowns in the three-legged frame of Fig. 1. These curves depict the effect of the variation of an elastic characteristic, at different points of the structure, on the values of three static unknowns, and they may be called the "variation lines." "Influence lines" might be a better term, but it would be unsuitable because it would disagree with the generally accepted meaning of this term. The influence line gives the value of a stress quantity for different positions of a unit load; the variation line, on the other hand, shows the change in a statically indeterminate quantity for different locations of the variation of an elastic characteristic, while the loading remains the same. For each static unknown there is a distinct variation line for every loading.

If an elastic characteristic undergoes a change i' at some location with an abscissa s on the length ds, the corresponding variation of the unknown is equal to the product of i', ds, and the ordinate of the variation line at the abscissa s. (There is also an additional constant factor, $\frac{1}{D}$, that can be brought into the expression for the variation line, as may be seen from Eq. 14.)

Variation lines are useful not only as a pictorial representation of the component parts of the variations contributed by different parts of the structure, but also as a means of determining, with some exercise of judgment, the probable variations of the unknowns, as distinct from the extreme values expressed by Eq. 15.

NOTE.—This paper by A. Hrennikoff, Assoc. M. Am. Soc. C. E., was published in January, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1942, by A. H. Finlay, Assoc. M. Am. Soc. C. E.; May, 1942, by Stefan J. Fraenkel, Jun. Am. Soc. C. E.; and October, 1942, by Joseph A. Wise, M. Am. Soc. C. E.

[9] Asst. Prof., Civ. Eng., Dept. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

[9a] Received by the Secretary December 21, 1942.

Professor Finlay has made use of these curves in his investigation of possible variations of stress quantities due to the frame cracking in the vicinity of points with high bending moments. In the structure considered, this effect proved to be very low; however, as the discusser correctly states, this may not be true in general.

The writer agrees with Mr. Fraenkel that practical applicability of the method considered may be confined to reinforced-concrete structures, in which the variations of the modulus of elasticity and of the moment of inertia may be quite substantial. In this connection, Mr. Fraenkel's suggestion to investigate the actual behavior of the structure as built, on the basis of tests of concrete specimens, taken from different parts of the structure, is quite appropriate, provided the test cylinders are made truly representative of the quality of concrete in the structure itself.

The writer prefers to differ with the discusser on the question of the suitability of varying the working stresses in reinforced-concrete structures as functions of sensitivity coefficients. In spite of Mr. Fraenkel's objections, the writer still thinks that even if such departure from the current practice might add to the complexity of design, the high values of sensitivity coefficients frequently occurring even in common structures, such as the one analyzed by Mr. Fraenkel, should make this measure quite in order, when designing structures of considerable importance.

Commander Wise has made an important contribution to the theory of the variation of statically indeterminate quantities by his illuminating interpretation of the significance of the expression for, what was termed herein, the "variation line." Eq. 45 is presented by Commander Wise in lieu of Eq. 15, as being applicable to structures with flexural stress. In this formula, M signifies the actual moments in the given structure with n static unknowns under the given loading, whereas m_a and δ_{aa}, respectively, represent the moments and the relative moment produced by a pair of stress quantities $X_a = 1$, in a modified structure with $(n - 1)$ unknowns, obtained from the given structure by removal of the restraint corresponding to X_a. This statement with regard to restraint should be understood in the following sense: If X_a signifies a bending moment at a point A, this modified structure should be assumed capable of a relative rotation, but not of a linear separation, of the two parts of it at this point; in other words, a pin is assumed to be inserted at the point A. On the other hand, if X_a signifies a shear, then the two parts of the structure are assumed capable of separating at the point A in the direction normal to the axis, while no tangential or angular relative movement is permitted at this point.

Eq. 45, proposed by Commander Wise, possesses the following advantages over the original Eq. 15:

1. It is simpler in form;
2. It illustrates the mechanical significance of the integrand, and thus can be computed by any suitable method, giving the values of M and m_a; and
3. It affords a better means of determining the null points of the integrand by equating to zero the expressions for M and m_a separately, rather than the product of these functions, which naturally has a more complex form.

Eq. 45 shows clearly why the variation line corresponding to the sum of two loadings acting simultaneously can be found by superposition of the two variation lines of the separate loadings. The reason for this relationship is that the expression m_a is the same in all three loadings, whereas the moment curve M of the combined loading is equal to the algebraic sum of the moment curves of the two separate loadings. The null points of the variation line of the combined loading, that come from m_a, are the same as in the separate loadings, whereas the null points produced by M are generally different in the three loadings considered.

The advantages of Eq. 45, demonstrated by Commander Wise and elaborated in the preceding paragraphs, make the inclusion of this equation into the theory of variation of static unknowns very desirable.

The counterpart of Eq. 45 applicable to structures with axial stresses has been given by Commander Wise in the form of Eq. 46. The significance of the normal stresses N and n_a in this equation is analogous to the significance of the moments M and m_a in Eq. 45.

Commander Wise suggests that the expression for the sensitivity coefficient K be modified by multiplying it by the ratio of the allowable and the actual unit stresses. The writer does not think that this modification would be of any advantage in the design, since the values of the variations need only be calculated for the stress quantities governing the design, in application to which the proposed modifying factor would always be not far from unity.

The writer is grateful to all the discussers for their contributions.

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DISCUSSIONS

UNUSUAL EVENTS AND THEIR RELATION TO FEDERAL WATER POLICIES

Discussion

BY W. G. HOYT, M. AM. SOC. C. E.

W. G. HOYT,[18] M. AM. SOC. C. E.[18a]—It was not, and is not, the intent of the writer to criticize any present water function or policy; the paper is factual and philosophical and not critical. Therefore, he will not attempt especially to point out "weak spots" as suggested by Mr. Sweet. As emphasized in the paper and discussions, water policies, at any one time, have depended in some immeasurable degree on climatic, economic, and social circumstances. When conditions change, policies respond. What may seem valid and constructive criticism currently, often looks ill-considered in retrospect. From the viewpoint of peacetime, Mr. Campbell may be justified in holding that federal water policies developed since 1930 "constitute one phase of a course of the federal government away from that American way of life which rests upon and is sustained by private initiative" and that many projects were extravagant and without economic justification.

From the point of view of peacetime, no one can yet say positively whether the projects were generally worth the cost. It is certain, however, that water planning and development have stood the Nation in good stead during the present emergency. By providing major sources of energy, many so-called extravagant developments have been the very foundation on which this country's enormous industrial expansion for war has been based. Factories were kept in operation during periods of depression, turning out equipment for use on what was often then considered "made work" incident to water, soil, and conservation projects. As a result, when it became necessary to establish army and navy bases in the four corners of the world, mobile equipment was available; many factories were tuned to produce and expand if necessary; and what was more important, many persons trained for peaceful employment in the Public Works Administration, Work Projects Administration, Civilian

NOTE.—This paper by W. G. Hoyt, M. Am. Soc. C. E., was published in February, 1942. *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1942, by Messrs. Edward H. Sargent, Dana M. Wood, James S. Sweet, and J. L. Campbell; and September, 1942, by Messrs. David J. Guy, and Albert S. Fry.

[18] Hydr. Engr. (Prin.), Water Resources Branch, U. S. Geological Survey, Washington, D. C.

[18a] Received by the Secretary December 23, 1942.

Conservation Corps, and other organizations, stepped into war activities without losing stride. This fortunate result is not necessarily attributable to the planners; nor does it follow that because peacetime policies proved their worth in wartime they were or are necessarily the best that could be devised. In fact, much evidence seems to indicate that major modifications in water policies are desirable.

The discussions by Messrs. Sargent, Wood, Guy, and Fry have added greatly to the treatment and development of the subject as a whole through inclusion of text and references outlining events and circumstances having to do with the water policies prior to the decade under consideration. The paper and discussions outline many of the events and circumstances by which the nation has arrived at its present water policy, a policy under which billions of dollars have been spent by federal, state, and local interests "without the benefit or direction of a consistent national water policy designed to produce the optimum coordinated development in the interests of all."[19] Now the Nation is at war, and until this war is won, there will be little modification of present policies. After the war is over, if policy makers agree with the National Resources Planning Board that "the present uncoordinated administration of laws and policies * * * cumulatively have not been sufficiently effective in producing properly balanced development of our resources for the people of the Nation, for the people of the regions or even for the people of certain individual basins,"[19] then they may well review some of the pertinent points brought out in the paper and discussions, parts of which are quoted herein.

Future policy makers must decide whether or not the best objectives will be accomplished by following the advisory leadership of the Water Resources Committee of the National Resources Board; or whether a similar committee should be vested with sufficient authority and so organized as to enable it to coordinate and direct future water policies; or whether the creation of a "hydraulic coordinating joint committee," as suggested by Mr. Sweet, "with a revolving chairmanship and executive powers" would be more appropriate. They must decide whether a basin project with all phases of that development under one federal agency, embracing a complete, unified type of organization for the planning of water-control projects and utilizing contributions of various sciences, such as hydrology, forestry, and agriculture, has more advantages than the procedure of planning by separate governmental agencies with separate functions and with Nation-wide scope, even though the planning is guided by a central committee.

In other words, should there be a Nation-wide group of authorities similar to the Tennessee Valley Authority, the diversified activities of which have been so clearly outlined by Messrs. Wood and Fry, or should developments for irrigation, power, flood control, and land-use continue to be administered by separate organizations, federal or state, the policies of which are fixed by law, or a combination of both? Should such groups (as Mr. Guy questions) embrace a federal water policy "shaped by the ebb and flow of unemployment" and use "soil and water conservation as a Constitutional peg for economic

[19] "National Resources Development—Report for 1943," National Resources Planning Board, Water Resources Section.

adjustment?" Future policy makers should avoid the danger that "any water policy may be literally pulled to pieces for no other purpose than to make legal and Constitutional pegs on which to hang a variety of new social and political philosophies." They should keep in mind that unusual meteorologic and hydrologic events, such as described in the paper, will recur and that in all probability the shaping of future policies will be affected by the occurrence and magnitude of these events. They should determine "whether the complete elimination of local contributions tends to make flood control a pork barrel proposition" or whether the present policy with respect to local contributions for irrigation development is the more equitable. They should determine whether or not the creation of many new federal agencies "to exercise the rights and powers yielded by states and communities" is wise and whether or not a federal water policy should be projected further into the fields of operation and management, "prompted, at times, by political rather than engineering considerations." These and many other pertinent questions raised in the discussions will necessarily have to be considered by those who will guide future policies.

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DISCUSSIONS

RESISTANCE OF SOFT FILL TO STATIC WHEEL LOADS

Discussion

BY JACOB FELD, M. AM. SOC. C. E.

JACOB FELD,[9] M. AM. SOC. C. E.[9a]—The correlation of the elastic analysis and field tests with observations of actual construction procedure and of results forms a valuable contribution to the subject of control and use of unstable ground areas. The superposing of a granular sand as a blanket over plastic or semifluid soils for the preparation of roadways is often part of heavy construction procedure in outlying areas. Mr. Pagon's analysis of the necessary thicknesses of granular materials will be of great aid. As reported by the writer[10] in 1940, similar results were found by trial experience in the construction of temporary roads in salt marshes. A blanket of sand 3 ft deep, placed by dumping from trucks, provided a roadbed which sustained heavily loaded construction trucks even though the adjacent ground could not be walked upon.

In very soft spots a layer of metal mesh or chain fabric pressed into the top surface of the sand will provide additional carrying capacity for the period in which the natural consolidation of the material makes it somewhat softer than the adjacent consolidated area. Such a method would aid in reducing the upkeep costs of runways placed upon plastic mud fills.

The action of the diaphragm formed in the contact zone, where the granular material and the finer grains of the original plastic mud intermix, was studied very carefully by Gen. M. C. Barberis, of the Italian Navy, from 1910 until quite recently. One of the reports easily available was presented at the 16th International Congress of Navigation, Brussels, Belgium, 1935, under the title, "Recent Examples of the Foundations of Quay Walls Resting on Poor Subsoil." This is the only report written by General Barberis in English, and makes reference to his various other more lengthy papers in Italian and French.

A summary of all of the studies made by General Barberis as listed by him for the writer in 1935, after a lengthy discussion following an inspection of the work in 1931, is as follows:

NOTE.—This paper by W. Watters Pagon, M. Am. Soc. C. E., was published in February, 1942 *Proceedings*.

[9] Cons. Engr., New York, N. Y.

[9a] Received by the Secretary April 29, 1942.

[10] *Proceedings*, Conference on Soil Mechanics, S. P. E. E., Purdue Univ., Lafayette, Ind., 1940, pp. 101–108.

(1) Docks built in Spezia, Italy, on a foundation of fluid subaqueous mud showed movements in accordance with the hypothesis and theories of Vierendeel and Rankine.

(2) The interposition of a layer of clean sand hydraulically placed between subaqueous mud and rock-fill foundation of the docks shows a resistance three times as great as would be expected from the Rankine theory in the work performed from 1912 to 1915, and six times the theoretical resistance was found in the work performed from 1930 to 1934.

(3) The great additional resistance resulting from the interposition of a clean, fine, sand layer is due to the large cohesion between that material and the underlying mud.

(4) When the sand layer is sufficiently extensive in area (say, 200 ft measured at right angles to the face of the dock) a thickness of 6 to 8 ft is sufficient to imprison the underlying clayey mud. After this condition had existed eighteen years, borings taken through the material indicated that the mud had not penetrated very far into the sand and that below the contact zone the mud had not been changed physically from the samples of uncovered or natural mud. Settlements of rock fill placed on the sand in 1912 had practically ceased in 1927.

(5) The artificial sand layer was dropped through 30 ft of water and spread very rapidly, and in a short time covered a large area of mud without disturbing the structure of the ancient mud layer.

(6) When the sand layer is 15 ft or more and extends for widths of 300 ft, the settlement of superimposed structure can be computed from the consolidation determined for the sand alone and without considering the effect on the underlying mud strata. Analyses indicate that a 15-ft depth of sand eliminated any possible excessive horizontal forces in the mud layer.

(7) In the work done between 1930 and 1934, sand placed 9 ft thick upon the mud supported loads of 2.71 kg per sq cm, or approximately 3 tons per sq ft. This was seven times the theoretical bearing capacity of the mud layer.

(8) In a section of the quay wall at the pierhead of one of the breakwaters built in 1934, with loads of 2.5 kg per sq cm placed on a sand layer 15 ft thick, there was no displacement of the foot of the quay wall, and no formation of a mud wave beyond the limits of the load. This indicated definitely that the sand layer takes the full load and distributes it to the mud without any local overstresses.

(9) In the breakwater built at Spezia, the layer of sand was 6 ft high and supported 2 km (1.24 miles) of breakwater. Similar construction of breakwaters was used at Kobe, Japan, except that the mud was disturbed by dumping rock from the surface into a trench dredged for the sand layer.

(10) Tidal changes of the range of 6 ft have not affected the stability of structures built on such subaqueous sand layers.

As Mr. Pagon states, the general idea of covering mud fill with granular materials is economically possible, but it must be done uniformly if satisfactory results are to be expected. Such uniformity is more easily obtained by placing the granular blankets by subaqueous methods, as was practiced by General Barberis. Developments of these studies to practical application for increasing the bearing values of mud fills not only for airport runways, but for the support of structures and roads, will be of great value to the profession, and as a contribution along this line, Mr. Pagon's paper is most welcome.

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DISCUSSIONS

NUMERICAL PROCEDURE FOR COMPUTING DEFLECTIONS, MOMENTS, AND BUCKLING LOADS

Discussion

BY MESSRS. A. A. EREMIN, AND MYRON L. GOSSARD

A. A. EREMIN,[29] ASSOC. M. AM. SOC. C. E.[29a]—An interesting method of making successive approximations for the computation of stresses and deformations is described in this paper. The method of successive approximations is exceedingly useful when sections of members carrying loads vary along the span length and when the sectional variation is difficult to express by a simple mathematical formula. The problems solved by the author clarify the practical value of the method.

A useful addition to the cases considered by Professor Newmark might be the case of a member resisting a bending moment applied at an intermediate section between the supported ends. This case may occur in the column that receives load applied through a bracket. Applying the method of successive approximations, the stresses and deformations in member AB, loaded with a considerable bending moment M at section C, were computed as shown in Fig. 36. The member was divided into six sections. The error in the computed restraining moment M_B at end B is about 15%. Obviously, to obtain greater precision in the restraining moment M_B, the member AB must be divided into a greater number of divisions. Therefore, it is not always safe to use five or six divisions as recommended by the author.

NOTE.—This paper by N. M. Newmark, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1942, by Bruce Johnston, Assoc. M. Am. Soc. C. E.; June, 1942, by M. S. Ketchum, Jr., Assoc. M. Am. Soc. C. E.; September, 1942, by Messrs. John B. Wilbur, Ralph W. Stewart, and Stefan J. Fraenkel; October, 1942, by Alfred S. Niles, Assoc. M. Am. Soc. C. E.; and November, 1942, by Camillo Weiss, M. Am. Soc. C. E.

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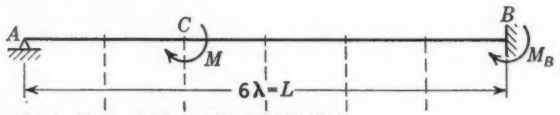
[29a] Received by the Secretary November 16, 1942.



(*a*) Deflection Due to Moment $M_B = 100$ at Right End

Moments	0	17	33	50	67	83	100
Distributed Angle Changes		−17	−33	−50	−67	−83	−100
Assumed Average Slope, Part (1)	−17	0	33	83	150	233	
Trial Deflection, Parts (1) and (2)	488	499	499	466	383	233	
Linear Correction to Deflection	488	407	326	245	164	81	0
Resultant Deflection	0	92	173	221	219	152	0

$$\text{Slope at right end} = (152 + 42)\frac{\lambda}{EI} = 194\frac{\lambda}{EI}$$

(*b*) Deflection Due to Moment $M = 100$

Moments	0	−17	+33	+50	+33	+17	0
Distributed Angle Changes		+17	−33	−50	−33	−17	0
Assumed Average Slope, Part (1)	−46	−63	−30	20	53	70	
Trial Deflection, Parts (1) and (2)	4	50	113	143	123	70	
Linear Correction to Deflection	4	3	2	1	....	....	
Resultant Deflection	0	47	111	142	123	70	

$$\text{Slope at right end} = (70 + 4)\frac{\lambda}{EI} = 74\frac{\lambda}{EI}$$

$$\text{Moment at left end} = M_B = \frac{74 \times 100}{194} = 38$$

FIG. 36.—RESTRAINING MOMENT PRODUCED BY MOMENT APPLIED AT A SECTION BETWEEN BEAM SUPPORTS

MYRON L. GOSSARD,[30] JUN. AM. SOC. C. E.[30a]—The numerical procedure for finding deflections and slopes of beams combines fundamental concepts of beam elasticity and geometry, simplicity and accuracy of method, and clearness as to beam action under load. These qualities, plus the fact that the procedure follows closely that of computing shear and moment diagrams, certainly will make it valuable to both structural engineers and engineering students. The application of the method is particularly effective in the analysis of continuous frameworks where it is necessary to determine certain beam constants and load constants. The author's procedure seems to possess all the advantages of the method of the column analogy when applied to beams, and gives a clearer picture of the beam action.[31] Also, the column analogy does not give the deflections directly as does the numerical procedure.

The writer has used a special application of the fundamental Newmark method to arrive at a procedure which, it is believed, is somewhat more convenient for finding stiffnesses, carry-over factors, fixed-end moments, and deflections of members of continuous frames—especially unsymmetrical members

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[30a] Received by the Secretary November 27, 1942.

[31] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, New York, N. Y. 1932, pp. 46–47.

that may be subdivided into segments that can be approximated by "standard" beam forms. The "standard" beam forms are those shown in Fig. 37, where the haunch curves may be either straight lines, half quadratic parabolas, or

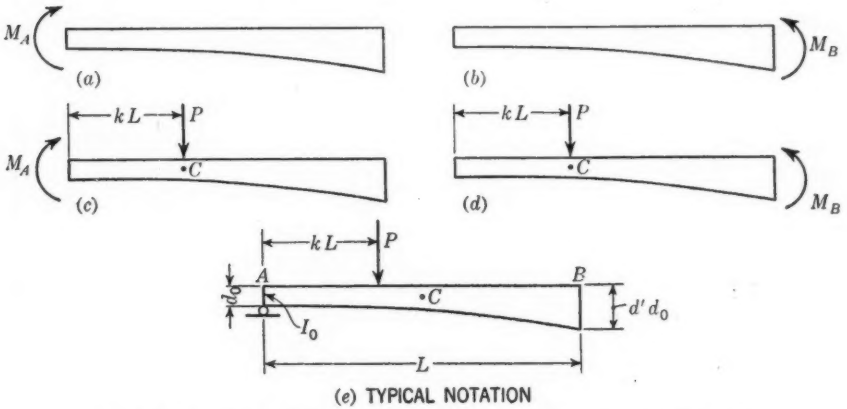

FIG. 37.—STANDARD BEAM FORMS (BEAM OUTLINES ONLY), WITH LOAD TYPES

half cubic parabolas, with vertexes at the shallow end of the beam. Following Professor Newmark's numerical procedure, sets of curves similar to Figs. 38

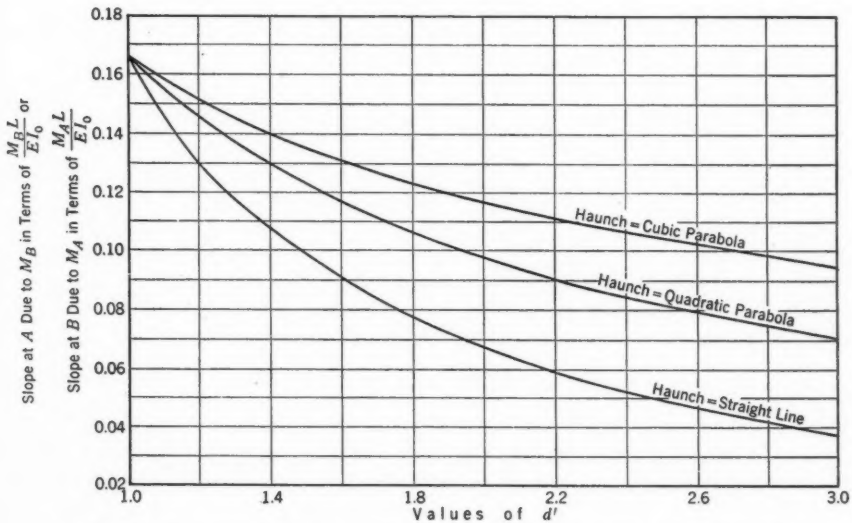

FIG. 38.—SLOPE AT ONE END OF A BEAM DUE TO A MOMENT APPLIED AT THE OTHER END (See Figs. 37(*a*) and 37(*b*))

and 39 were drawn for each case, using twelve equal divisions of beam length, and based on the assumption that, at any section, the moment of inertia varie

as the $\frac{5}{2}$ power of the beam depth. This assumption seems to the writer to be valid and sufficiently accurate for both structural steel (plate girder or I-beam) and reinforced concrete (rectangular or T-beam) construction, inasmuch as the exponent must lie between 2 and 3 for these types and some error in this respect does not appreciably affect the results of analyses.[32] Fig. 38 is for the case of a moment applied at one end or the other end of a beam (Figs. 37(*a*) and 37(*b*)),

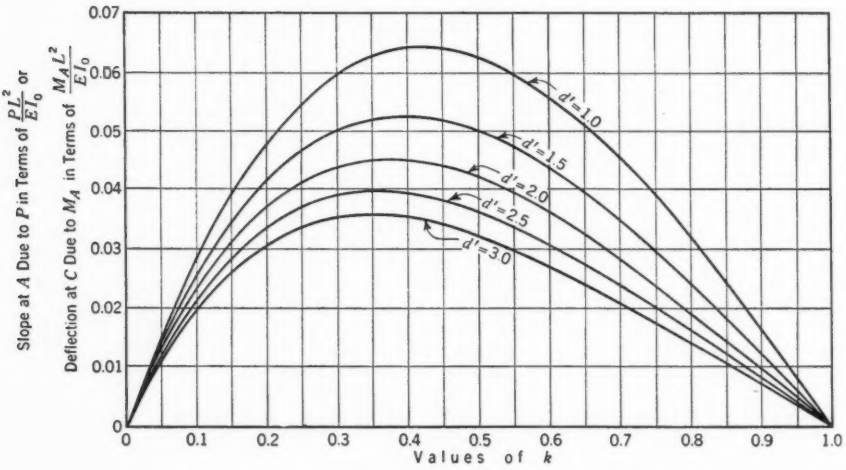

FIG. 39.—SLOPE AT END A DUE TO A CONCENTRATED LOAD (See Fig. 37(*c*)); OR DEFLECTIONS ALONG THE BEAM DUE TO A MOMENT APPLIED AT END A (See Fig. 37(*a*))
(Haunch = Quadratic Parabola)

giving the slope at A due to M_B, in terms of $\frac{M_B L}{E I_o}$ (or the slope at B due to M_A, in terms of $\frac{M_A L}{E I_o}$). For the purposes of a complete solution, similar curves (not published) were constructed for the slope at A due to M_A, and for the slope at B due to M_B. Fig. 39 is typical of curves giving the slope at A due to a concentrated load P (Fig. 37(*c*)), in terms of $\frac{P L^2}{E I_o}$. The curves of Fig. 39 also give the deflections along the beam due to M_A (Fig. 37(*a*)), in terms of $\frac{M_A L^2}{E I_o}$, by Maxwell's law of reciprocal displacements. Curves similar to those of Fig. 39 (not published) were prepared for straight-line and third-degree parabolic haunches; and for the slope at B due to a concentrated load P (Fig. 37(*c*)), the latter set of curves also giving the deflections along the beam due to M_B (Fig. 37(*b*)). Briefly, six sets of curves are needed for a complete solution of a case of the type in Fig. 40, representing:

[32] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, New York, N. Y., 1932, pp. 3–5 and 169–171.

Case	Description
(*a*)	The slope at A due to M_A
(*b*)	The slope at B due to M_A, which is equal to the slope at A due to M_B
(*c*)	The slope at B due to M_B
(*d*)	The deflection at C due to M_A, which is equal to the slope at A due to a concentrated load P at C
(*e*)	The deflection at C due to M_B, which is equal to the slope at B due to a concentrated load P at C
(*f*)	The deflection at beam-center due to a concentrated load P

An example of what may be encountered and what may be done with the aid of such data is illustrated in Fig. 40, from which all the information necessary for a complete analysis of the beam as a part of a continuous frame is derived. However, if deflection is not important or if "highly accurate" deflection curves and influence lines are not required, a considerable amount of the work can be eliminated. Fig. 40(*a*) shows the beam to be made up of three elastic segments, L_1, L_2, and L_3, with an assumed inelastic segment at each end. The trapezoidal moment diagram for each elastic segment as a part of the whole is divided into two triangular diagrams; then end slopes and deflections for each segment due to each of its triangular moment diagrams are taken from the appropriate curves of the types of Figs. 38 and 39. The end slopes are added at each segmental junction or "joint" to become the "equivalent concentrated angle changes" from which the average slopes and string deflections of the joints are obtained as in the fundamental procedure. To the string deflections are added the segmental deflections at selected points between the joints (at center points in the example) to obtain the deflection curve. Figs. 40(*b*) and 40(*c*) show the calculations and deflection curves for $M_A = 100$ and $M_B = 100$, respectively. In Fig. 40(*d*) the simple beam deflection curves are combined to give curves of deflection for end moments of 100 with the far ends fixed These may be used as influence lines for fixed-end moments. Fig. 40(*e*) is included to illustrate the procedure for finding the influence line for beam-center deflection which is always close to the maximum deflection in both simple or continuous beams. By proper combinations of Figs. 40(*b*), 40(*c*), and 40(*e*) the center deflection for any load condition on the continuous beam may be found.

To complete the discussion there follow the calculations for moment-distribution constants, fixed-end moments, and simple beam center deflection for dead and live loads shown in Fig. 40(*f*). In the calculations involving influence lines and distributed loads, the area under a curve is approximated by finding the area under a parabola passing through three points, which area (A) is given by

$$A = \frac{\lambda}{6}(a + 4b + c) \qquad (33)$$

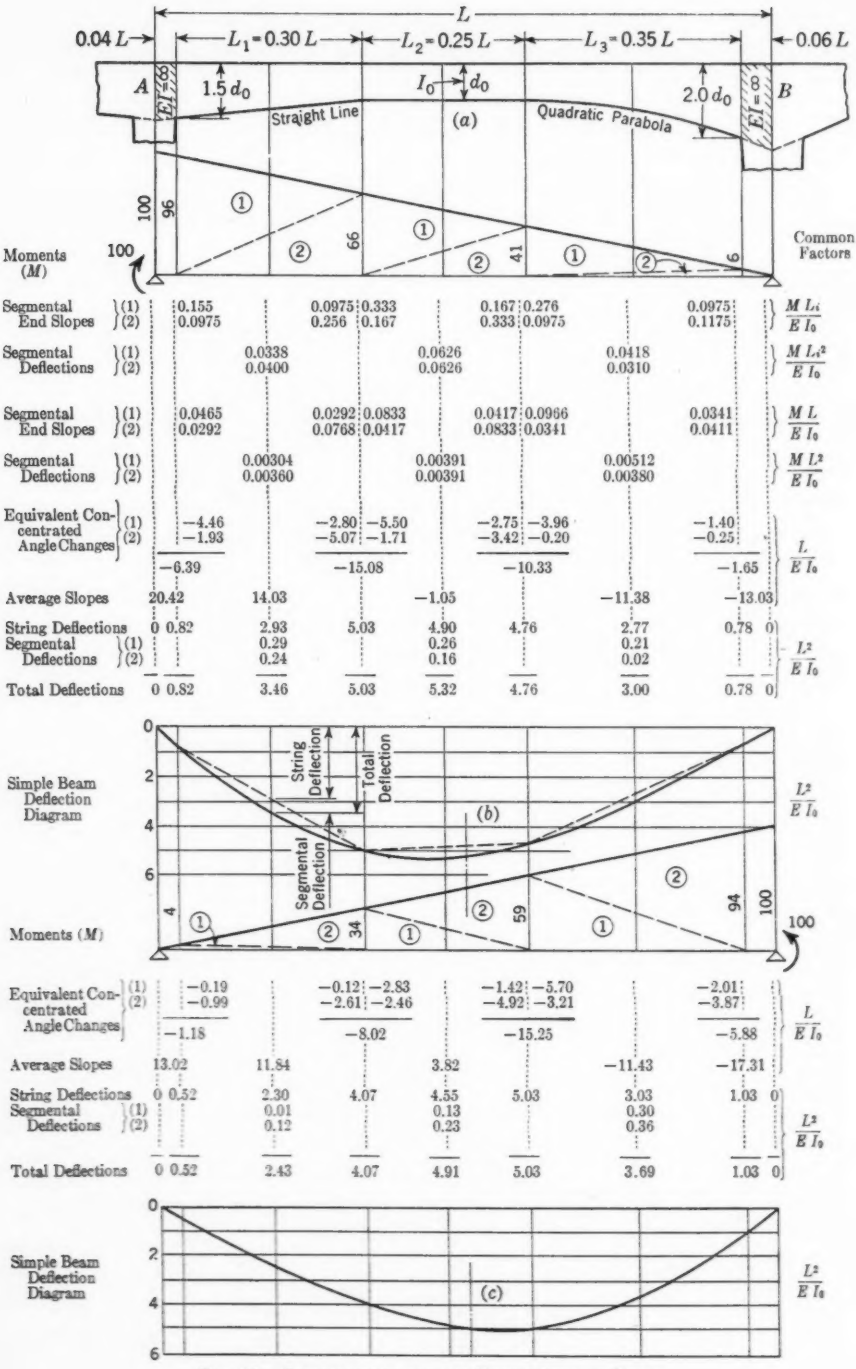

FIG. 40.—COMPUTATIONS FOR AN UNSYMMETRICAL BEAM

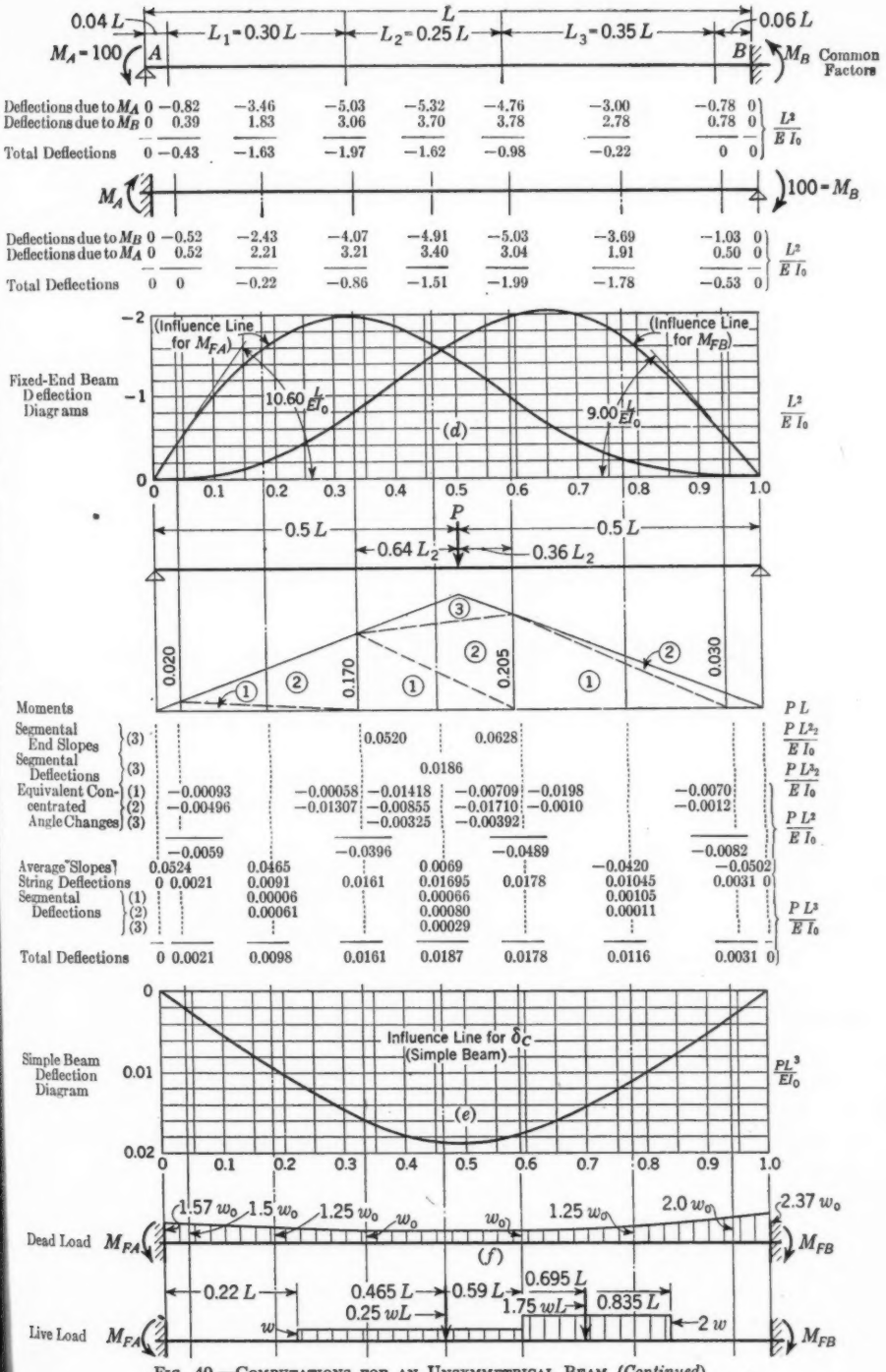

FIG. 40.—COMPUTATIONS FOR AN UNSYMMETRICAL BEAM (*Continued*)

In Eq. 33, a and c are bounding ordinates of the horizontal length λ, and b is the center ordinate.

The beam constants are as follows:

Assuming positive rotations as clockwise, the rotation at one end of a simple beam, due to unit clockwise moment applied at that end or the opposite end, is—

$$\phi_{AA} = + 0.2042 \frac{L}{E I_o} \quad \ldots\ldots (34a)$$

$$\phi_{BA} = - 0.1303 \frac{L}{E I_o} \quad \ldots\ldots (34b)$$

$$\phi_{BB} = + 0.1731 \frac{L}{E I_o} \quad \ldots\ldots (34c)$$

and

$$\phi_{AB} = - 0.1302 \frac{L}{E I_o} \quad \ldots\ldots (34d)$$

In Eqs. 34, the second letter of the subscript denotes the end of the beam where the moment is applied, Eqs. 34a and 34b referring to Fig. 40(b) and Eqs. 34c and 34d to Fig. 40(c).

Assuming that positive moments produce tension in the bottom fibers of a beam, the carry-over factor is the ratio of the moment at one end to the moment applied at the other end, when one end or the other is fixed:

$$r_{AB} = \frac{\phi_{AB}}{\phi_{BB}} = \frac{-0.1302}{0.1731} = - 0.752 \quad \ldots\ldots (35a)$$

and

$$r_{BA} = \frac{\phi_{AB}}{\phi_{AA}} = \frac{-0.1302}{0.2042} = - 0.637 \quad \ldots\ldots (35b)$$

For example, Eq. 35a yields the carry-over factor at end A when end B is fixed, and Eq. 35b, the carry-over factor at end B when end A is fixed. For the moment stiffness S_M, the moment at end A required to produce unit rotation at end A when end B is fixed, is

$$(S_M)_A = \frac{1}{\phi_{AA} - r_{AB}\,\phi_{AB}} = 9.40 \frac{E I_o}{L} \quad \ldots\ldots (36a)$$

Similarly for the moment at end B required to produce unit rotation at end B when end A is fixed—

$$(S_M)_B = \frac{1}{\phi_{BB} - r_{BA}\,\phi_{AB}} = 11.1 \frac{E I_o}{L} \quad \ldots\ldots (36b)$$

From Figs. 40(d) and 40(f), the fixed-end moments (M_F) are computed as follows.

Dead Load.—

$$\frac{0.30\,L}{6}[1.5\,(0.43) + 4\,(1.25)\,(1.63) + 1.97]\,w_o = 0.538\,w_o\,L$$

$$\frac{0.25\,L}{6}[1.97 + 4\,(1.62) + 0.98]\,w_o = 0.393\,w_o\,L$$

$$\frac{0.35\,L}{6}[0.98 + 4\,(1.25)\,(0.22) + 0]\,w_o = 0.121\,w_o\,L$$

$$0.04\,L\,(1.53)\,(0.22)\,w_o = 0.013\,w_o\,L$$

$$\text{Total} = 1.065\,w_o\,L$$

$$(M_F)_A = 1.065\,w_o\,L \times \frac{L}{10.60} = 0.100\,w_o\,L^2\,(-) \ldots\ldots\ldots (37a)$$

and

$$\frac{0.30\,L}{6}[0 + 4\,(1.25)\,(0.22) + 0.86]\,w_o = 0.098\,w_o\,L$$

$$\frac{0.25\,L}{6}[0.86 + 4\,(1.51) + 1.99]\,w_o = 0.371\,w_o\,L$$

$$\frac{0.35\,L}{6}[1.99 + 4\,(1.25)\,(1.78) + 2\,(0.53)]\,w_o = 0.697\,w_o\,L$$

$$0.06\,L\,(2.18)\,(0.27)\,w_o = 0.035\,w_o\,L$$

$$\text{Total} = 1.201\,w_o\,L$$

$$(M_F)_B = 1.201\,w_o\,L \times \frac{L}{9.00} = 0.134\,w_o\,L^2\,(-) \ldots\ldots\ldots (37b)$$

Live Load.—

$$\frac{0.37\,L}{6}[1.75 + 4\,(1.83) + 0.98]\,w = 0.620\,w\,L$$

$$\frac{0.245\,L}{6}[0.98 + 4\,(0.40) + 0.05]\,2\,w = 0.215\,w\,L$$

$$0.25\,w\,L\,(1.62) = 0.405\,w\,L$$

$$1.75\,w\,L\,(0.46) = 0.805\,w\,L$$

$$\text{Total} = 2.045\,w\,L$$

$$(M_F)_A = 2.045\,w\,L \times \frac{L}{10.60} = 0.193\,w\,L^2\,(-) \ldots\ldots\ldots (38a)$$

and

$$\frac{0.37\,L}{6}[0.31 + 4\,(1.18) + 1.99]\,w = 0.433\,w\,L$$

$$\frac{0.245\,L}{6}[1.99 + 4\,(1.96) + 1.34]\,2\,w = 0.913\,w\,L$$

$$0.25\,w\,L\,(1.51) = 0.378\,w\,L$$

$$1.75\,w\,L\,(2.0) = 3.50\,w\,L$$

$$\text{Total} = 5.224\,w\,L$$

$$(M_F)_B = 5.224\,w\,L \times \frac{L}{9.00} = 0.580\,w\,L^2\,(-) \ldots\ldots\ldots (38b)$$

From Figs. 40(*e*) and 40(*f*) the simple beam center deflections are computed as follows.

Dead Load.—

$$\frac{0.30\,L}{6}\,[1.5\,(0.0021) + 4\,(1.25)\,(0.0098) + 0.0161]\,w_o = 0.00341\,w_o\,L$$

$$\frac{0.25\,L}{6}\,[0.0161 + 4\,(0.0187) + 0.0178]\,w_o = 0.00453\,w_o\,L$$

$$\frac{0.35\,L}{6}\,[0.0178 + 4\,(1.25)\,(0.0116) + 2\,(0.0031)]\,w_o = 0.00478\,w_o\,L$$

$$0.04\,L\,(1.53)\,(0.0011)\,w_o = 0.00007\,w_o\,L$$

$$0.06\,L\,(2.18)\,(0.0016)\,w_o = 0.00021\,w_o\,L$$

$$\text{Total} = 0.01300\,w_o\,L$$

$$\delta_c = 0.01300\,w_o\,L \times \frac{L^3}{E\,I_o} = 0.0130\,\frac{w_o\,L^4}{E\,I_o}\,(+) \quad \ldots\ldots\ldots\ldots (39)$$

Live Load.—

$$\frac{0.37\,L}{6}\,[0.0112 + 4\,(0.0179) + 0.0178]\,w = 0.00620\,w\,L$$

$$\frac{0.245\,L}{6}\,[0.0178 + 4\,(0.0138) + 0.008]\,2\,w = 0.00662\,w\,L$$

$$0.25\,w\,L\,(0.0187) = 0.00467\,w\,L$$

$$1.75\,w\,L\,(0.0145) = 0.0254\,w\,L$$

$$\text{Total} = 0.0429\,w\,L$$

$$\delta_c = 0.0429\,w\,L \times \frac{L^3}{E\,I_o} = 0.0429\,\frac{w\,L^4}{E\,I_o}\,(+) \quad \ldots\ldots\ldots\ldots (40)$$

Lack of space prevents the inclusion of the various curves prepared for use in the discussion of Fig. 40. In other respects, however, the writer has entered into more detail than is necessary in most applications of the numerical procedure because the emphasis of the Newmark paper is on a technique. Technique may rightfully assume prominence in problems such as this.

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DISCUSSIONS

VISCOSITY AND SURFACE TENSION EFFECTS ON V-NOTCH WEIR COEFFICIENTS

Discussion

BY G. H. HICKOX, M. AM. SOC. C. E.

G. H. HICKOX,[23] M. AM. SOC. C. E.[23a]—This paper is a genuine contribution to knowledge in the field of flow measurement, and the author is to be congratulated on his effort in bringing this information before the profession. A number of years ago, the writer became interested in the effect of viscosity on weir discharge because of an inquiry as to the proper weir discharge coefficient to be used in the measurement of printer's ink. He assisted in the preparation of the experimental apparatus on which the University of California results, referred to by the author, were obtained. It is gratifying to see that the ensuing experiments have been put to good use.

In the past, practically all information on weir discharge coefficients has been obtained from experiments using water. The effects of viscosity, surface tension, and inertia have been combined into a single coefficient that also includes the geometry of the approach channel and the roughness of the weir surface, as well as any other variables which might affect the discharge. Attempts have been made to evaluate the effect of the geometry of the approach channel by including the velocity of approach with the measured head. The results have not been entirely satisfactory. The most consistent results have been obtained when the dimensions of the approach channel have been such that the velocity of approach was negligible. A notable exception is the use of the ratio of the head on the weir to the height of weir crest above the channel bottom in the case of a rectangular weir. The effects of viscosity and surface tension have been almost totally neglected.

It should be recognized that, after the author's demonstration of the use of R and W as suitable parameters, the treatment is empirical. It is quite possible that the form of the function indicated by Eq. 10 might be consider-

NOTE.—This paper by Arno T. Lenz, Assoc. M. Am. Soc. C. E., was published in March, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1942, by A. A. Kalinske, Assoc. M. Am. Soc. C. E.; and September, 1942, by Messrs. Glen N. Cox and F. J. Germano, Emery H. Willes, and Ralph W. Powell.

[23] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[23a] Received by the Secretary November 30, 1942.

ably different from that developed in Eq. 13. It is conceivable that several different functions might exist which would satisfy the experimental data equally well. Extrapolation of these functions outside the experimental range would doubtless give varying results. For this reason, extrapolation of the results of the experiments beyond the range of the experimental data is unwarranted and should be made with caution or not at all.

The use of V-notch weirs as measuring devices need not be confined solely to fluids of low viscosity, such as water or lubricating oils. These weirs may find application in the handling of heavy inks, paints, and sugar solutions, including molasses, to mention only a few examples. Additional experiments with fluids of high viscosity would determine the effects of viscosity, and possibly surface tension, over a much wider range. This would be advantageous in the application to fluids of lower viscosity since it would remove the uncertainties existing at the end of the present useful range.

The effects of viscosity and surface tension on V-notch weirs are paralleled more or less closely by the effects on small ogee weirs. Tests on small-scale models of ogee spillways are being made in large numbers, and the results are being transferred to the much larger prototype structures by means of Froude's law, neglecting the effects of viscosity and surface tension. Such a transference of results assumes that the coefficient for the prototype is the same as for the model. Experimental verification of this assumption is almost entirely lacking, and the evidence available does not point to any definite conclusions. From experiments on three different scale models of San Gabriel Dam No. 1, Paul Baumann,[24] M. Am. Soc. C. E., found that the discharge coefficient increased with the size of the model. A comparison[25] of measured discharges over an eastern dam with model tests made by the Bureau of Reclamation on a 1 : 72 scale model showed that the prototype coefficients were smaller than those of the model. A. H. Gibson[26] and D. Thoma[27] have presented similar results. No definite information is available to explain the differences that have been found. Although it is possible that prototype discharge measurements may be in error, it seems much more likely that the neglected effects of viscosity and surface tension in small models would be found to be responsible.

In spite of the writer's own caution against extrapolation of results, he would like to inquire whether any work done by the author has indicated a possible answer to this problem. The question is not academic. The matter of predicting spillway discharges from model tests becomes important on those rivers that are completely canalized and that also develop power at the navigation dams. Under such conditions, an accurate measurement of discharge is useful for reservoir regulation and power production. The usual rating stations are drowned out by downstream dams, and the discharges can be determined only by calibration of the turbines and spillway gates. The easiest way of

[24] "Design and Construction of San Gabriel Dam No. 1," by Paul Baumann, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1595.

[25] "Conformity between Model and Prototype, A Symposium: Spillway Coefficients at an Eastern Dam," by G. H. Hickox, *Proceedings*, Am. Soc. C. E., October, 1942, p. 1334.

[26] "The Use of Models in Hydraulic Engineering," by A. H. Gibson, *Water and Water Engineering*, Winter 1934, pp. 697–704.

[27] *Transactions*, Hydraulic Lab. of The Technical University of Munich, Germany, No. 2, 1928.

making the spillway calibration is by means of model tests, if it can be established that they are reliable. It is possible that studies similar to those reported in the paper would make it possible to evaluate the effects of viscosity and surface tension on the results of small-scale model tests.

Corrections for *Transactions:* In March, 1942, *Proceedings*, page 358, change the last two lines to read: "*Head-Coefficient Data and Curves.*—The experimental coefficients for the 90° V-notch for oils A and G are shown in Fig. 7. The * * *"; page 359, line 11, change "data" to "series"; page 359, line 12, change "showed" to "show"; page 359, line 33, change "*b*" to "*L*"; and page 365, line 35, change, "$C = 0.56$" to "$C - 0.56$."

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DISCUSSIONS

ENTRAINMENT OF AIR IN FLOWING WATER
A SYMPOSIUM

Discussion

BY WARREN DELAPP, JUN. AM. SOC. C. E.

WARREN DELAPP,[24] JUN. AM. SOC. C. E.[24a]—It is safe to state that Mr. Hall's paper is by far the most comprehensive to have been published on this subject, and should prove a valuable addition to the present meager knowledge of the behavior of high velocity flow.

The apparent inconsistency notable in Fig. 10 may be caused by two factors in addition to errors in the data. The first and most important of these is Mr. Hall's attempt to relate the percentage of air in the flow to a parameter of questionable significance, $\frac{V^2}{g R_c}$, erroneously designated the Froude number. In addition to the objections that might be raised to the use of so fictitious a term as R_c, attention must be called to the fact that the Froude number $\frac{V}{\sqrt{g D}}$ is an indication of the ratio of the kinetic energy of the flow to its potential energy; it loses its significance completely if the linear dimension becomes a function of the channel width.

A second reason for the scattering of the points in Fig. 10 is the author's attempt to find a single equation generally applicable to the wide variety of flow conditions occurring in each channel. An examination of the data reveals that, for the same value of $\frac{V^2}{g R_c}$, the percentage of air is consistently greater for retarding flow than for accelerating flow. Flow behavior at changes in grade—vertical and horizontal curves—introduces new variables so that Eq. 24*a* (representing a mean of all these conditions when K and K_1 are evaluated for a given channel) can scarcely be more than a crude approximation when applied to any channel not similar in plan and profile to the one for which it was derived.

NOTE.—This Symposium was published in September, 1942, *Proceedings*.

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[24a] Received by the Secretary December 9, 1942.

A function of perhaps greater significance may be developed by considering only points at which the flow was approximately in equilibrium, eliminating data taken when the flow was rapidly accelerating or retarding and observations near horizontal or vertical curves. These data are of little value in the solution of the fundamental problem, and tend only to obscure the true form of the function. All data from the Kittitas flumes are acceptable on this basis, although, obviously, normal flow was not attained on the slope of 0° 04′, since it was less than critical. However, the rate of retardation was apparently sufficiently low to have little effect on the air entrainment. Normal flow was approximated only near Station 2 + 50 and Station 2 + 75 in the Hat Creek chute, and in Rapid Flume from Station 3 + 50 to Station 4 + 00.

As Mr. Hall has implied (see heading, "Analyses of Results"), although the amount of air entrained in the water should depend primarily on the Froude number, it also should depend to some extent on the channel roughness, since the latter will affect the intensity of agitation of the flow for a given value of **F**. The actual form of the function appears to be

$$\frac{Q_{aw}}{Q_w} = K\,\mathbf{F} \qquad (39)$$

in which Q_{aw} is the combined quantity of air and water, **F** the Froude number $\frac{V}{\sqrt{g\,y_o}}$, and K a dimensionless number whose value depends on the roughness of the channel. Fig. 17 indicates that K has a value of 0.178 for Kittitas 1 and Rapid Flume, and a value of 0.227 for Kittitas 2 and 3, the value for Hat Creek chute apparently lying between these two. Values of **F** where $\frac{Q_{aw}}{Q_w} = 1$ apparently indicate the point at which air entrainment begins for the particular channel roughness, and values less than unity evidently have no meaning.

The form of Eq. 39 indicates a simple relationship between y_o, the depth of flow, and q_w, the water discharge per unit width of channel, for

$$\frac{Q_{aw}}{Q_w} = \frac{b\,y_o\,V}{b\,q_w} \qquad (40a)$$

and

$$\frac{y_o\,V}{q_w} = K\frac{V}{\sqrt{g\,y_o}} \qquad (40b)$$

so that

$$y_o = K^{\frac{2}{3}}\sqrt[3]{\frac{q^2_w}{g}} \qquad (41)$$

Substituting the values of K from Fig. 17, Eq. 41 becomes, for Kittitas 1 and Rapid Flume:

$$y_o = 0.316\sqrt[3]{\frac{q^2_w}{g}} \qquad (42a)$$

and, for Kittitas 2 and 3,

$$y_o = 0.372\sqrt[3]{\frac{q^2_w}{g}} \qquad (42b)$$

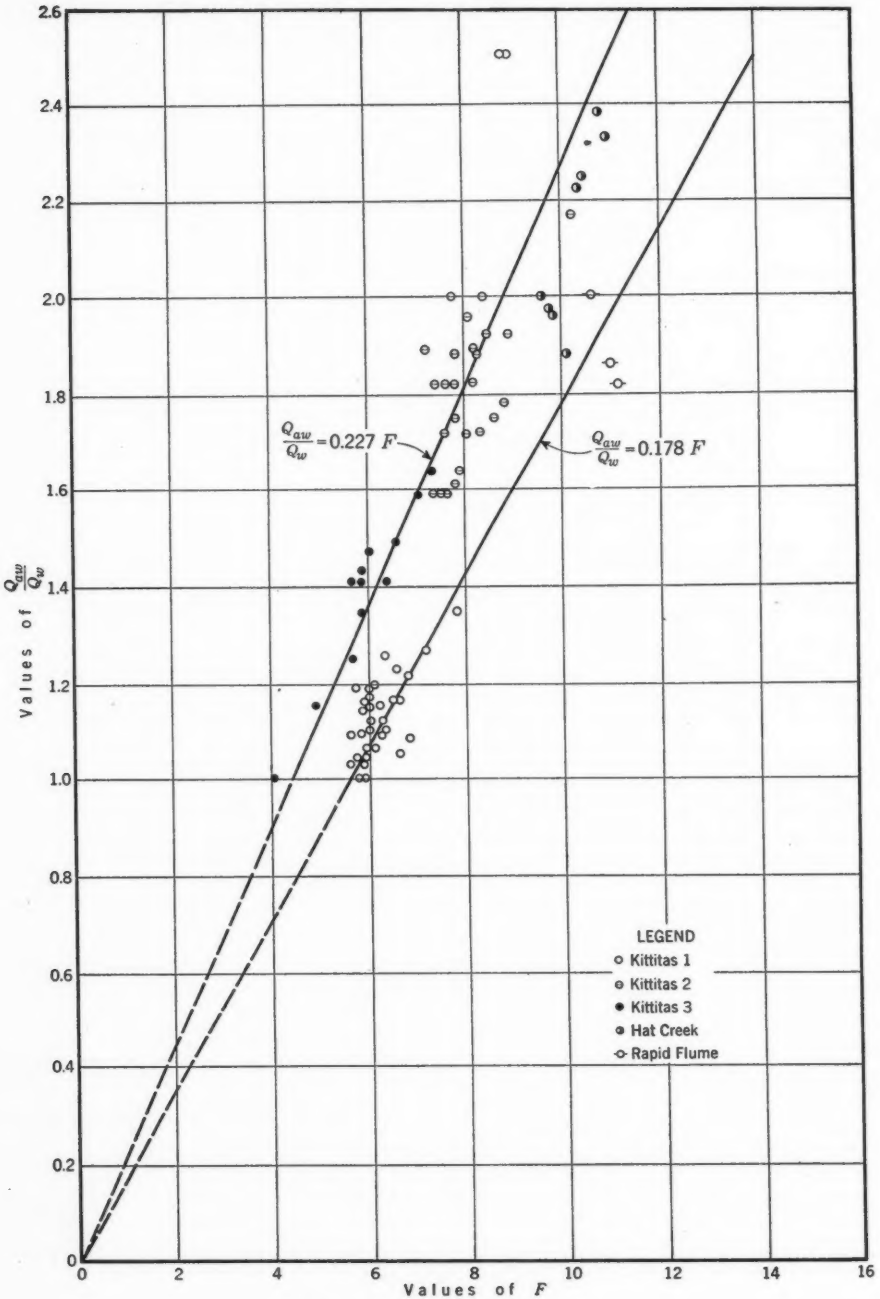

FIG. 17.—RELATION OF $\frac{Q_{aw}}{Q_w}$ TO F

These functions are plotted in Fig. 18 and show a close agreement with the actual measured depths of flow from Mr. Hall's data.

It remains to evaluate K in terms of the channel roughness, the information available being scarcely sufficient for that purpose. There would appear to be some correlation between the values of K and values of n as computed by the author, but that these n-values are the same as would be found for sub-

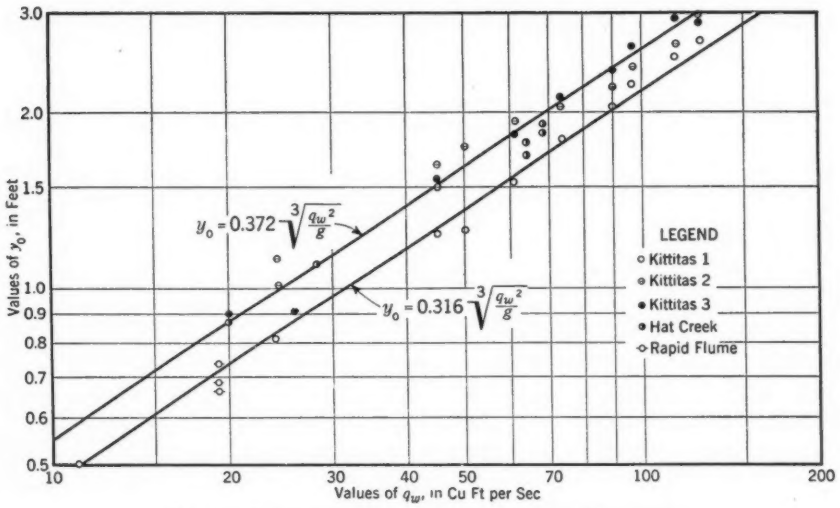

FIG. 18.—RELATION OF DEPTH TO RATE OF WATER DISCHARGE

critical velocities is by no means certain. Indeed, the validity of the Manning formula for high velocities is scarcely more than an assumption on the part of Mr. Hall. For a given slope and roughness the Manning formula may be reduced to the form

$$V = K R^m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

A plot of V against R_o from the author's data reveals that any value of m from $\frac{1}{2}$ to $\frac{2}{3}$ fits the data equally well, the range of values not being sufficiently great to establish the slope of the line with any degree of certainty. Neither is the fact that n is relatively constant for a given channel of great significance, as the values of R_o are so close to unity that a change in the exponent makes little change in the value of n.

However, in spite of these facts, Eq. 41 should prove adequate for the design of straight chutes at uniform grades, the relatively small variation of the coefficient making its selection of no greater uncertainty than that normally encountered in the choice of n. For nonuniform flow conditions, and in the design of horizontal and vertical curves, the data and observations presented by Mr. Hall will undoubtedly prove of great value.

Correction for *Transactions:* In September *Proceedings*, 1942, page 1145, Fig. 13, change "α" to "β," also, change "0" to "0.01" and "0.01" to "0.015" on the ordinate scale; in Fig. 16, page 1149, change "β" to "θ," and in Eq. 36, change "$-$" to "$+$."

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DISCUSSIONS

CONFORMITY BETWEEN MODEL AND PROTOTYPE A SYMPOSIUM

Discussion

BY MESSRS. C. I. GRIMM, AND JOE W. JOHNSON

C. I. GRIMM,[16] M. AM. SOC. C. E.—A number of acceptable formulas have been developed for use in determining the crest profile of an overflow concrete spillway. The profile of the crest of the spillway reported by Messrs. Stevens and Cochrane comes within the limits of these formulas and, therefore, may be considered as a conventional shape. Computations for a thin jet were made as a guide to design. These showed that, with the gate in the downstream position, the jet followed quite close to the crest profile, but that with the gate in the upstream position it departed considerably. It was not considered practicable, however, to design the profile so that there would be no departure in this case. These calculations would raise doubt as to whether the positive pressures indicated by the model with the gate in the upstream slot (Table 7) were correct.

The prototype studies seem to prove that the model did not give correct results in regard to pressures on the crest and indicate that the difference between the model and prototype may be due to non-similitude of atmospheric pressure.

With the gate in the downstream slot on the prototype, little negative pressure was found. The upstream slot is for emergency use only in the event that the downstream slot is out of use for repairs. It is not intended, or necessary, that gates in these slots be used for fine regulation, but rather in a well-opened position. As a practical operating condition, only small negative pressures have existed on the spillway crest at the dam discussed by the authors. Examination has shown the concrete to be undamaged thereby, although extensive damage to the piers and some damage to the crest have occurred just downstream from the vertical gate slots.

NOTE.—This Symposium was published in October, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by A. E. Niederhoff, Assoc. M. Am. Soc. C. E.

[16] Formerly with U. S. Engr. Dept. (Mr. Grimm died on December 1, 1942.)

Comparison of prototypes with models is an important step in hydraulic investigations that has been entirely inadequate in scope, to date, and the study by Messrs. Stevens and Cochrane is a valuable contribution. A model study of spillway crests under controlled atmospheric pressure would be timely and of decided value as a research project to supplement present knowledge.

JOE W. JOHNSON,[17] ASSOC. M. AM. SOC. C. E.[17a]—The various papers included in this Symposium indicate that a wide variety of hydraulic problems is susceptible to investigation by means of models. All of the papers are important contributions to a field in which the literature is rather meager.

Of particular interest to the writer were the papers by Mr. Hickox and by Mr. Soucek on spillway coefficients. In the investigations described in these two papers, a period of five years was required to obtain the various data. Mr. Hickox describes prototype observations and compares them with model studies that were made when the structure was in the design stage. On the other hand, Mr. Soucek describes model studies and compares the data with past observations on an existing structure. The sequence of the observations is unimportant for giving valuable information on conformity between model and prototype. Model studies made prior to the construction of a prototype give valuable assistance in obtaining the best design, but model studies made of existing structures are more likely to be academic in character. Somewhat between these two limits was a model study conducted at the University of California at Berkeley, Calif., on the spillway crest of the Upper Narrows Dam on the Yuba River. This investigation (58)[17b] was intended primarily to provide (1) a check on the assumed discharge curve of the spillway, and (2) measurements of the profile of the nappe at various discharges for the purpose of predicting the point where the water would strike the stream bed at the base of the dam.

The Upper Narrows Dam, intended for flood control and as a debris barrier, is of the constant-angle arch type with a curved crest (Fig. 63). The length of the spillway crest is divided into twelve sections by piers 5 ft thick. Except for the two end sections, all sections have a length of 100 ft from center line to center line of piers. To take the normal flows over the dam a center section, 4 ft lower than the other sections, is provided. The length of this section is 189.4 ft, excluding piers, and the remaining crest length, 908.4 ft. The elevation of the depressed section is 527 ft, and the elevation of low flow in the original stream channel was 291.2 ft.

From a consideration of the model and prototype measurements on the Keokuk Dam on the Mississippi River (21*a*), the designers assumed the relationship between the weir coefficient C (computed from Eq. 13) and the energy head shown in Fig. 64. The following discharge formula was then determined from a logarithmic plot of head versus discharge per foot of crest, the discharge

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[17a] Received by the Secretary December 17, 1942.

[17b] Numerals in parentheses, thus: (58), refer to corresponding items in the Bibliography, which is appended to the Symposium as a separate unit, and to the end of the discussion in this issue.

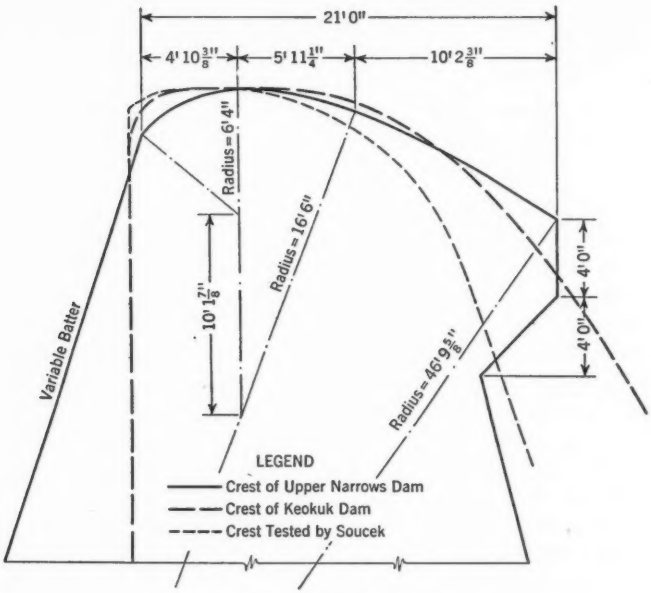

FIG. 63.—COMPARATIVE CREST DESIGNS

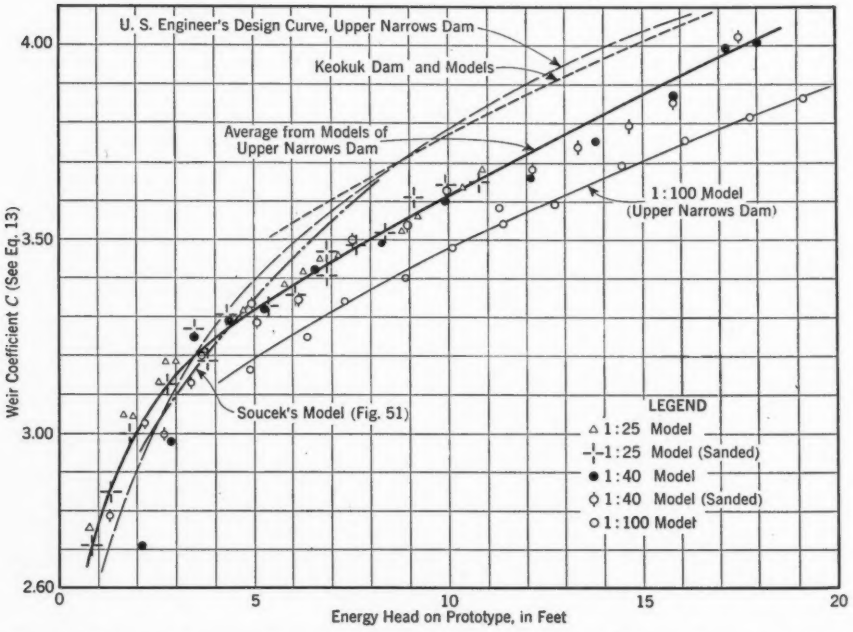

FIG. 64.—RELATION BETWEEN WEIR COEFFICIENT AND ENERGY HEAD ON VARIOUS DAMS AND MODELS

being determined from Eq. 13 and the weir coefficient curve of Fig. 64:

$$Q = 2.62\, H^{1.66} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Fig. 63 shows a comparison of the crest profiles of the Keokuk Dam and Upper Narrows Dam.

To check Eq. 15 and provide data on the profile of the nappe, three models were constructed using scales of 1 : 25, 1 : 40, and 1 : 100 (see Fig. 65). The two larger scales were used primarily for the determination of the discharge characteristics of the spillway at the dam, and the 1 : 100 model was used to determine the profile of the nappe.

The models were tested in a flume 6 in. wide and 3 ft high, equipped with a forebay and baffles. Discharge was measured by means of an orifice meter in the supply line leading from a constant-head tank. All models were of smooth, varnished wood, 6 in. wide, and approximately 2 ft high. They thus represented only the upper part of the dam (depending on the scale), as shown in Fig. 65. The heads used in the analysis included the measured depth above the crest and the velocity head in the approach channel and, therefore, represented the energy head. Observations were taken with heads varying from 0.030 ft to 0.450 ft. Because of the relatively strong surface tension forces at the heads below 0.03 ft, the nappe would cling to the face of the model—a condition which probably would not exist on the prototype at a corresponding head. A high degree of turbulence in the approach channel prevented accurate readings of the head for heads greater than 0.450. All runs were made with the nappe completely aerated by means of a 1-in. pipe through which air could flow to the underside of the nappe.

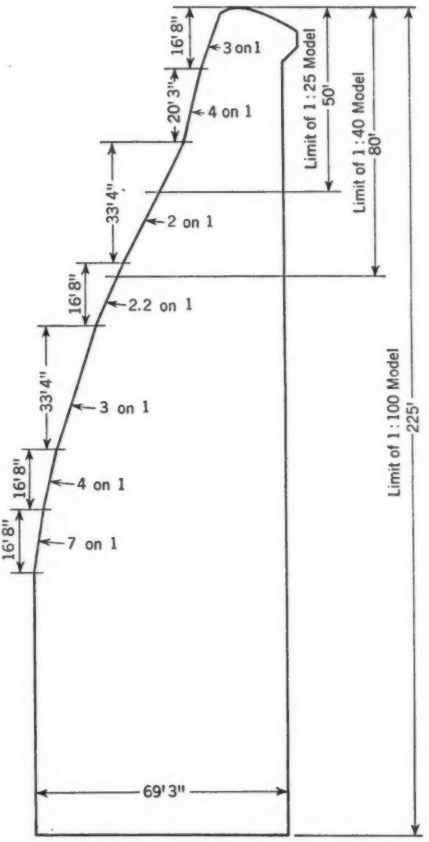

FIG. 65.—CROSS SECTION OF 1 : 100 MODEL
(Distances Given in Prototype Values)

To determine whether frictional forces had any significant effect on the weir characteristics over the range of flow under investigation, tests were conducted on the 1 : 25 and 1 : 40 models with a smooth, varnished surface and with a sand-rough surface. The roughened surface was obtained by

sprinkling sand, which passed the 100-mesh sieve and was retained on the 200-mesh sieve, on the freshly varnished surface of the model.

Fig. 64 shows the relation between the weir coefficient and the energy head for the various models. The values for the 1 : 25 and 1 : 40 models, whether with a smooth or sand surface, agree very well and can be represented by an average curve. In addition, the results from the 1 : 100 model, which are consistently lower than the larger models, are shown in Fig. 64.

By using the average curve drawn through the experimental points for the 1 : 25 and 1 : 40 models (Fig. 64), values of discharge were computed for various assumed values of head. A plot of discharge versus head on logarithmic paper gave the relationship:

$$Q = 2.79\, H^{1.62} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

For comparative purposes, the data of Mr. Soucek are also of interest. Thus, the mean curve through the experimental points in his head-coefficient relation obtained by model studies (Fig. 51) is also plotted in Fig. 64. This curve follows quite closely the designer's curve based on the Keokuk experiments, and such agreement is to be expected when it is considered how similar the profiles of Keokuk Dam and the dam discussed by Mr. Soucek are to each other (Fig. 63). A more definite measure of the agreement between the various studies is obtained if the head-discharge relation shown in Fig. 55 is plotted on logarithmic paper. If the discharge per foot is considered instead of the total discharge, this plot yields the equation

$$Q = 2.85\, H^{1.62} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Comparing Eq. 17 with Eqs. 15 and 16, it is found that below a head of about 3 ft the various equations give practically the same discharge. For a head of 10 ft, Eqs. 15 and 17 give discharges that are about 3% greater than the Upper Narrows Dam (Eq. 16). For almost all design problems this agreement can be considered as well within the limits of error to which certain basic design data, such as the "maximum" spillway discharge, can be estimated.

Bibliography.—

(58) **"Models of Upper Narrows Dam Spillway,"** by W. M. Stubblefield and J. P. Henck, thesis presented to the University of California in December, 1939, in partial fulfilment of the requirements for the degree of Bachelor of Science (unpublished).

(21a) **"Experiments on Discharge over Spillways and Models, Keokuk Dam,"** by F. A. Nagler and A. Davis, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 810.